INSTITUTO POLITÉCNICO DO PORTO

INSTITUTO SUPERIOR DE ENGENHARIA

ON THE ANALYSIS OF (UN)TRUE ROOT MEAN SQUARE MEASUREMENT

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PREFACE

This document is the result of the work developed by Anna Maria Hulak, under the supervision of Mário Ferreira Alves, in the aim of the ERASMUS European Program for students' exchange. The experimental part of the project was carried out in the Instrumentation and Measurements Laboratory, Electrical Engineering Department of ISEP, between April and June 1999.

The main objectives of the project were:

- to distinguish the two kinds of root mean square (RMS) measuring instruments - "conventional" (mean-based, average responding) or true RMS (TRMS) and to give a basic description of their distinct working principles;
- to determine the error committed by conventional multimeters when measuring non-sinusoidal signals (by analytical calculus);
- to show how the problem of limited instrument bandwidth can affect measurements;
- to develop an experimental analysis with several "conventional" and TRMS instruments in order to validate the analytical results.

First, the need for TRMS measuring instruments is justified. The second chapter makes a very simple flashback to show why "conventional" multimeters are suited for measuring the RMS of pure sinusoidal voltages or currents. In order to prove that "conventional" multimeters do not measure the TRMS of non-sinusoidal signals, a mathematical analysis is undertaken in the third chapter. This problem is overcome with TRMS instruments (mainly voltmeters, ammeters and digital sampling oscilloscopes) that may have different working principles. This subject is discussed in chapter 4.

The fifth chapter describes the problem of limited bandwidth and presents a comparison between supplier's (bandwidth) specifications and experimental results. Chapter 6 describes all the experimental analysis about true and untrue RMS measurements, for several waveforms. An analytical versus experimental analysis is also carried out. Finally, chapter 7 gives an overview of the entire project, including problems that appeared during the experimental phase, practical remarks, and what could be the future sequence of this work.

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1. INTRODUCING THE INTEREST OF TRMS MEASURING INSTRUMENTS

An alternated signal has several important features, such as the period, frequency, maximum amplitude (peak value), peak-to-peak value and root mean square (RMS) value. From this set, the RMS value is certainly the most used characteristic. For this purpose, the most used instrument is the multimeter, due to its cost/benefit ratio.

Before semiconductors devices (diodes, transistors, tiristors, etc.) becoming commercially available, every electric circuit was exclusively constituted by resistances, capacitors and coils. These electrical components have the property of not distorting an electrical signal, i.e., if the electrical current is sinusoidal, the voltage will have the same format. Due to this factor, these components are called passive or linear:

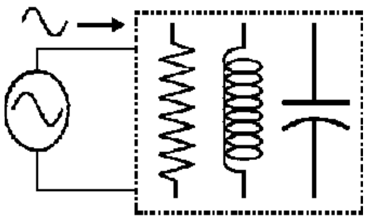

Figure 1: Circuit with linear components (sinusoidal voltage and current) ([Fluke, 1998])

This is the case of heating resistance heaters, induction motors and tungsten or halogen lamps.

Although, in the major part of current electric and electronic systems, other components are also included (like diodes, triacs, zeners) that provoke that the voltage and current circuit are not sinusoidal in every point in the circuit, even if the power source delivers a sinusoidal voltage or current. For this reason, these components are called non-linear:

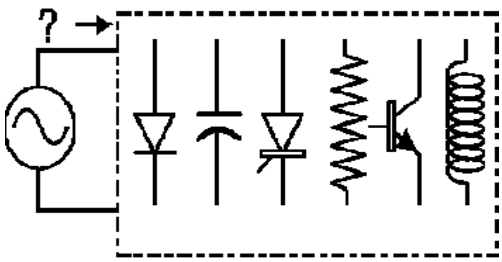

Figure 2: Circuit with non-linear components (sinusoidal voltage but non-sinusoidal current) ([Fluke, 1998])

The so-called “conventional” multimeters are average (more common) or (less common) peak responding instruments, basing their RMS measurement in the signal mean value. For this reason, they are only suited for sinusoidal signals, justifying the development of a new kind of multimeter - the True RMS (TRMS) multimeter. Since these multimeters do not base their RMS measurements in the mean value, they are suited for any kind of waveform.

As an example, if we measure the current passing in a non-linear load both with a TRMS multimeter and also with a non-TRMS (“conventional”) multimeter, we will certainly get different values (example with current probes):

Figure 3: TRMS and non-TRMS current probes ([Fluke, 1998])

Now, if we want to measure the current in one (ore more) computers or in a motor drive, again we are facing non-linear loads. So, in spite the power source supplying a sinusoidal voltage, the current will not be sinusoidal:

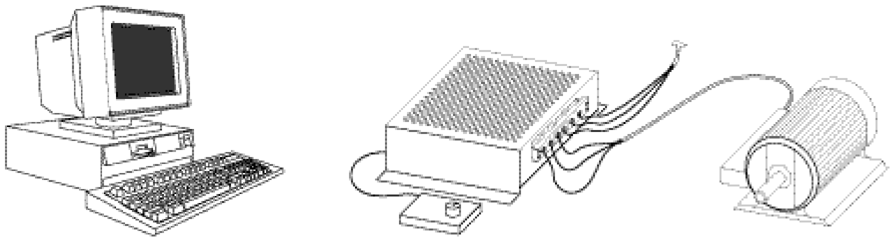

Figure 4: Computer and motor drive (non-linear loads) ([Fluke, 1998])

In the next picture, the same multimeter (Fluke 867 Graphical Multimeter) displays two (different) values for the current RMS– one (left) true and the other (right) untrue (mean-based):

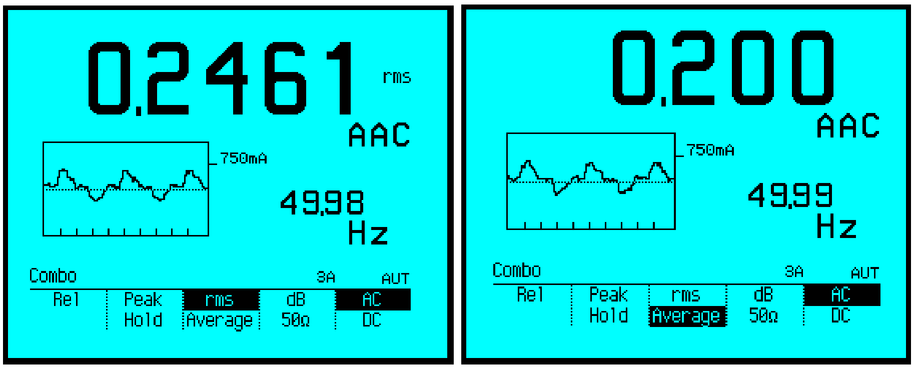

Figure 5: True and untrue RMS values ([Fluke, 1998])

As may be observed, the current waveform is not sinusoidal, resulting in an 18.4% difference between TRMS and mean-based RMS values.

2. “CONVENTIONAL” MULTIMETERS ARE GOOD FOR SINUSOIDAL SIGNALS

First hand, it is very important to understand how average-responding multimeters measure the RMS of sinusoidal voltages or currents.

2.1. Characteristics of a Sinusoidal Signal

Maximum Amplitude - U_m

Also known as maximum value or peak value, the maximum amplitude is the maximum instantaneous value reached by the signal (voltage - $\mathbf{U_m}$, or current - $\mathbf{I_m}$):

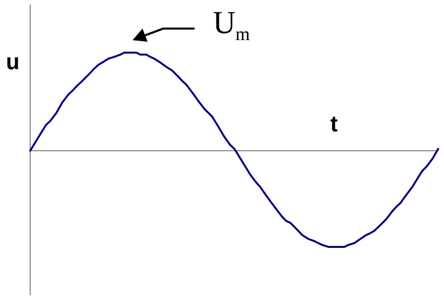

Figure 6: Maximum amplitude of a sinusoidal signal ([Alves, 1999])

Both positive and negative maximum amplitudes may be considered. In the case of an alternated signal, they have the same value.

Instantaneous Value - u(t)

The instantaneous value of a sinusoidal quantity - **u** – may be mathematically represented as a function of time - **t**:

$$u(t) = U_m.\sin(\omega t)$$

where ω represents the angular speed in radians per second - **rad/s**. The relationship between the angular speed, the frequency and the period is the following:

$$\omega = 2\pi.f = 2\pi \ / \ T$$

If we consider a vector **<u>U</u>**, of size $\mathbf{U_m}$, rotating with a speed ω, the instantaneous value will be the vertical projection of that vector:

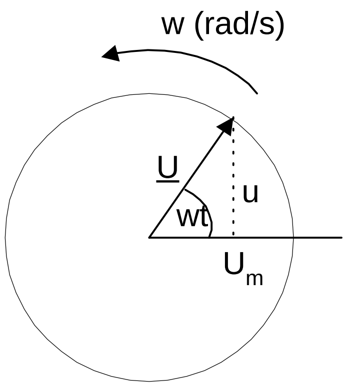

Figure 7: Instantaneous value as a projection of rotating vector ([Alves, 1999])

It is easy to confirm in the previous graphic that the following equation is true:

$$u(t) = U_m.\sin(\omega t)$$

Period - T and Frequency - f

Since an AC signal repeats itself periodically (cyclically), one of its fundamental characteristics is the value of the time interval between repetitions (or cycles), i.e., the **period** - **T**, which is measured in seconds **– s**:

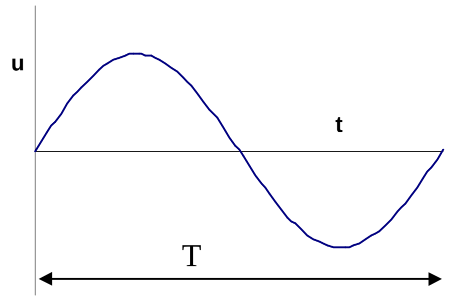

Figure 8: Period of a sinusoidal signal ([Alves, 1999])

It is very common to use another characteristic of sinusoidal signals, that is directly related to the period – the **frequency** - **f**. This quantity represents the number of cycles that occur in a second and its unit is the **Hertz** - **Hz**.

The relationship between the frequency and the period is the well known:

$$f = \frac{1}{T}$$

Mean Value - U_{mean}

The mean (average) value of a signal is defined as:

$$U_{mean} = \frac{1}{T}\int_0^T u(t)dt$$

If the signal is sinusoidal, the mean value is obviously null (the positive area of the signal is equal to its negative area). On the other hand, many times it is interesting to know the mean value of the rectified signal (not null), as we are going to show forward in the text.

Root Mean Square Value - U

The root mean square value of a sinusoidal (AC) signal is equal to the value of a constant (DC) signal if they produce the same heating power (*Joule's Effect*) in a pure resistance, for a given time period:

$$U = \sqrt{\frac{1}{T}\int_0^T (u(t))^2\, dt}$$

This relationship comes from the fact that the power in a resistor is proportional to the square of the voltage (or the square of the current), discarding a constant (R), both for AC and DC signals.

In the particular case of sine waves, the RMS value is $\sqrt{2}$ smaller than the maximum value (this will be proved later), independently of the frequency (*Figure 9*):

$$I = \frac{I_m}{\sqrt{2}} \approx 0.7 \times I_m \qquad \text{e} \qquad U = \frac{U_m}{\sqrt{2}} \approx 0.7 \times U_m$$

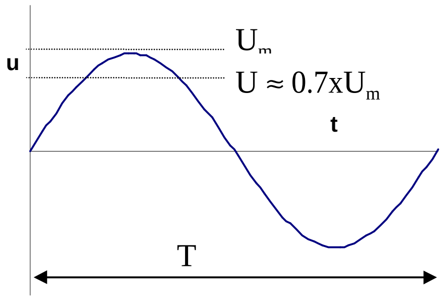

Figure 9: RMS and maximum values of a sinusoidal signal ([Alves, 1999])

Please note that:

- The ratio of $\sqrt{2}$ between the maximum and RMS values only applies to AC. For other waveforms, the relationship is different (this will be proved later).
- Multimeters always indicate a RMS value, when measuring sinusoidal voltages or currents.
- When an AC value is given, this will always be a RMS value, unless other is explicitly mentioned. For instance, in electric power transportation, if a line has a voltage of 400 kV, that is a RMS value.

Nevertheless, the maximum value may be more important than the RMS value, in certain cases, like the project of electric insulation. For example, the maximum admitted value for a multimeter might be 1000 V for DC and 750 V for AC (since a RMS value of 750 V roughly corresponds to a maximum value of 1000 V).

2.2. The Need for Rectification

Since the major part of "conventional" voltmeters and ammeters are mean-based, their indication for a sine wave would be, in principle, zero. To overcome this problem, a possible solution is to make the current unidirectional. That is achieved by means of a rectifier. If both half-cycles (positive and negative) are rectified, we are facing a full-wave rectification (4 diodes):

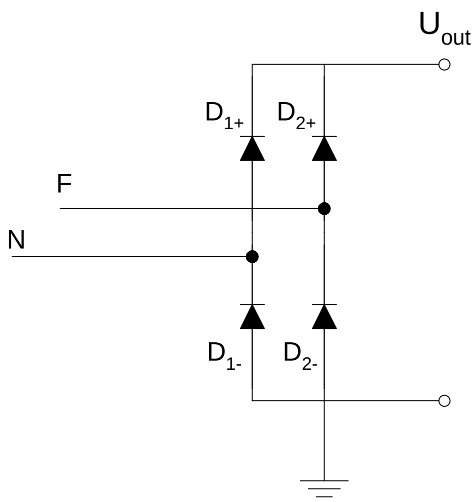

Figure 10: Full-wave rectification ([Alves, 1999])

Input and output signals will have the following format:

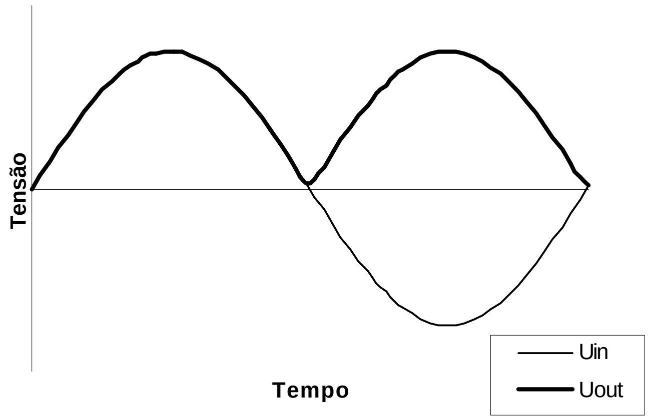

Figure 11: Input and output waveforms ([Alves, 1999])

Measurements may also be achieved using half-wave rectification. In this case, only one diode is used, but only half of the signal (power) is obtained at output:

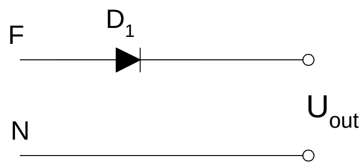

Figure 12: Half-wave rectification ([Alves, 1999])

The waveforms would be like this:

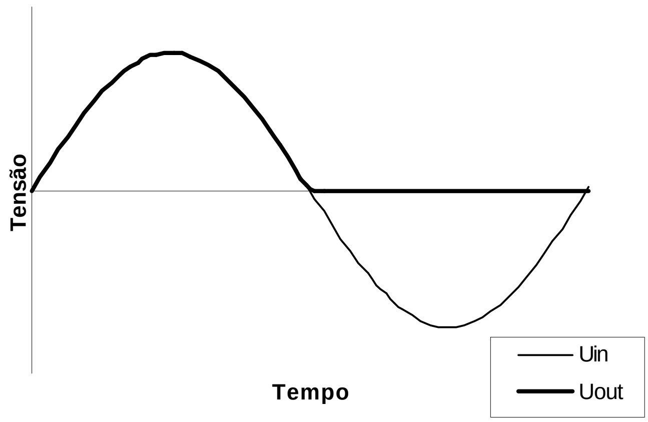

Figure 13: Input and output waveforms ([Alves, 1999])

Taking into account the mathematical definitions of mean and RMS values, these kind of "conventional" instruments (mean-based) will have to determine the RMS through the multiplication of the mean value by a certain quantity (form factor):

- In full-wave rectification: $U = 1{,}11.U_{mean}$
- In half-wave rectification: $U = 2{,}22.U_{mean}$

Note: these relationships will be proved forward in the text.

Obviously, the indicated value only is correct if the signal is purely sinusoidal. As will be proved in chapter '3. "Conventional" Multimeters are not good for Non-Sinusoidal Signals', the resulting RMS will be a value proportional to the signal mean.

3. “CONVENTIONAL” MULTIMETERS ARE NOT GOOD FOR NON-SINUSOIDAL SIGNALS

In this section, we will prove that “conventional” multimeters do not give a correct measurement of RMS, when the input signal is non-sinusoidal. That proof is achieved by analytical calculus for several waveforms.

3.1. “Sine” Wave

Pure Alternated

Lets remember again the RMS mathematical definition:

$$U^2 = \frac{1}{T}\int_0^T (u(t))^2.dt \Rightarrow U = \sqrt{\frac{1}{T}\int_0^T (u(t))^2.dt}$$

For a sine wave:

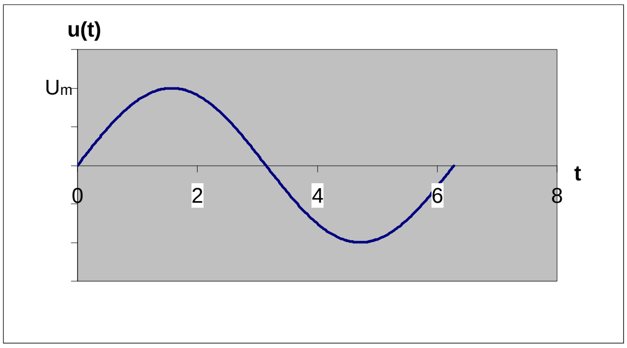

Figure 14 Sinusoidal wave

The RMS value is, by definition:

$$U^2 = \frac{1}{T}\int_0^T U_m^2.\sin^2(wt).dt$$

Considering that,

$$\sin^2(wt) = \frac{1}{2}(1+\cos(2wt))$$

We get,

$$U^2 = \frac{U_m^2}{T}\int_0^T \frac{1}{2}.(\ 1-\cos(2wt)\ ).dt$$

$$U^2 = \frac{1}{2}.\frac{U_m^2}{T}\left(\int_0^T 1.dt - \int_0^T \cos(2wt).dt\right)$$

$$U^2 = \frac{1}{2}.\frac{U_m^2}{T}\left[t - \frac{1}{2w}.\sin(2wt)\right]_0^T$$

$$w = {}^{2p}\!/_{T}$$

$$U^2 = \frac{1}{2}.\frac{U_m^2}{T}.T$$

$$U^2 = \frac{U_m^2}{2} \Rightarrow U = \frac{U_m}{\sqrt{2}}$$

As the mean (average) value of the sinusoid is zero, all “conventional” multimeters consider the mean of half a period:

$$U_{mean} = \frac{2}{T}\int_0^{T/2} U_m.\sin(wt).dt \Leftrightarrow U_{med} = U_m.\frac{2}{T}.\left[\frac{-\cos(wt)}{w}\right]_0^{T/2}$$

$$U_{mean} = \frac{2.U_m}{T}\left(-\frac{1}{w}\left(\cos(\frac{wT}{2}) - \cos(0)\right)\right)$$

$$U_{mean} = U_m.\frac{2}{T}.\frac{2}{w} \Leftrightarrow U_{med} = U_{m.}\frac{2}{T}.\frac{2}{2p/T}$$

So, the mean value is:

$$U_{mean} = \frac{2}{p}.U_m$$

For a sinusoidal signal, the relationship between the RMS and mean values is the following

$$U = \frac{U_m}{\sqrt{2}} \wedge U_{mean} = \frac{2}{p}.\sqrt{2}.U$$

$$U = \frac{p}{2\sqrt{2}}U_{mean} \Rightarrow U = 1{,}11 \times U_{mean}$$

“Conventional” multimeters usually base their RMS measurements in the mean value of the rectified signal (sometimes in the maximum or peak value). These multimeters just multiply the mean value by (form factor):

$$p/2\sqrt{2} \approx 1.11$$

to get the RMS value.

From the above, “conventional” multimeters are suited for measuring the RMS of sinusoidal signals.

Variable DC Component

If we have a sine wave with a variable DC component:

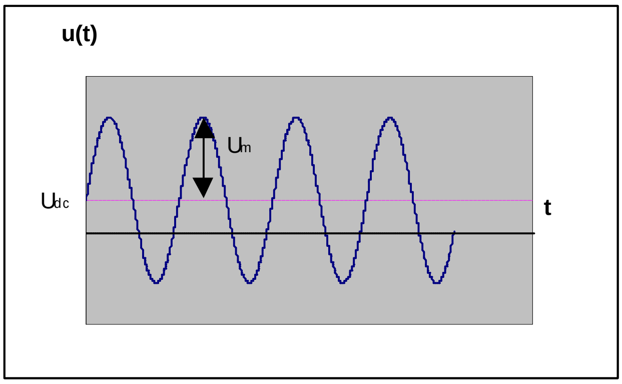

Figure 15 Sinusoidal wave with DC component

The RMS value can be determined:

$$U^2 = \frac{1}{T}\int_0^T \left(U_m \sin(wt) + U_{DC}\right)^2 dt \Leftrightarrow$$

$$U^2 = \frac{1}{T}\int_0^T \left(U_{DC}^2 + 2.U_{DC}.U_m \sin(wt) + U_m^2 \sin^2(wt)\right) dt \Leftrightarrow$$

$$U^2 = \frac{1}{T}\int_0^T U_{DC}^2 dt + \frac{1}{T}\int_0^T 2.U_{DC}.U_m \sin(wt) dt + \frac{1}{T}\int_0^T U_m^2 \sin^2(wt) dt \Leftrightarrow$$

$$U^2 = \frac{U_{DC}^2}{T}\int_0^T dt + 0 + \frac{U_m^2}{T}\int_0^T \sin^2(wt) dt \Leftrightarrow$$

$$U^2 = U_{DC}^2 + U_{AC}^2 \Leftrightarrow U^2 = U_{DC}^2 + \frac{U_m^2}{2}$$

As we can see, the DC component may be separated from the sinusoidal signal. This will be generalized for any periodic waveform in'5.1. RMS DC and AC Components'.

Variable Triggering Angle

This kind of signal is used in several applications, namely to regulate the light (power) of a lamp:

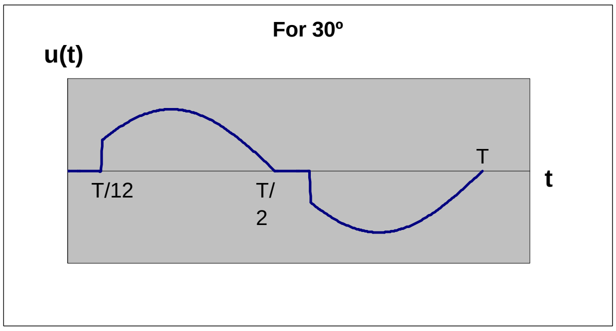

Figure 16 Sinusoidal wave triggered at 30º

Mathematically, the RMS value is easy to determine if we consider a sinusoidal signal starting in a certain angle. For the particular case of a 30º:

$$U^2 = \frac{2}{T}\int_{T/12}^{T/2} U_m^2 . \sin^2(wt).dt$$

$$U^2 = \frac{U_m^2}{T}\left[t - \frac{1}{2w}\sin(2wt)\right]_{\frac{T}{12}}^{\frac{T}{2}}$$

$$U^2 = \frac{U_m^2}{T}\left[\frac{5T}{12} - \frac{1}{2w}\left(\sin(wt) - \sin(\frac{wT}{6})\right)\right]$$

$$U^2 = U_m^2\left[\frac{5}{12} - \frac{1}{4p}\sin(\frac{p}{3})\right] \Leftrightarrow U^2 = 2.U_m^2\left[\frac{5}{12} + \frac{\sqrt{3}}{8p}\right]$$

$$U = U_m\sqrt{\frac{5}{12} + \frac{\sqrt{3}}{8p}} \Leftrightarrow U = 0.69U_m$$

The mean value is:

$$U_{mean} = \frac{2}{T}\int_{T/12}^{T/2} U_m \sin(wt)dt$$

$$U_{mean} = \frac{2U_m}{T}\left[-\frac{1}{w}\left(\cos(\frac{wT}{2}) - \cos(\frac{wT}{12})\right)\right]$$

$$U_{mean} = \frac{U_m}{p}\left[1 + \frac{\sqrt{3}}{2}\right]$$

A “conventional” multimeter would indicate:

$$U_{volt} = \frac{p}{2\sqrt{2}}U_{mean} \Rightarrow U_{volt} = \frac{1}{2\sqrt{2}}\left[2 + \frac{\sqrt{3}}{2}\right]U_m$$

The resulting error can be determined:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left[1 + \frac{\sqrt{3}}{2}\right]\frac{1}{\sqrt{\frac{5}{12} + \frac{\sqrt{3}}{8p}}} = 0.946$$

$$U_{volt} = 94.6\%U$$

In this case, the error is 5.4%.

We can do the same for a 90º triggering angle:

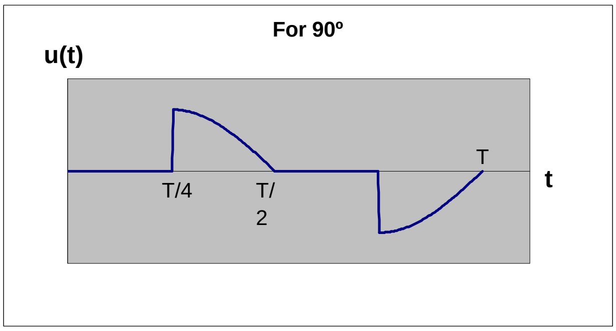

Figure 17 Sinusoidal wave triggered at 90º

Several calculus methods can be used, like considering a half-sinusoid, but the result is obviously the same. Using the previous used method:

$$U^2 = \frac{2}{T}\int_{T/4}^{T/2} U_m^2 \sin^2(wt)dt$$

$$U^2 = \frac{U_m^2}{T}\left[t - \frac{1}{2w}\sin(2wt)\right]_{\frac{T}{4}}^{\frac{T}{2}}$$

$$U^2 = \frac{U_m^2}{T}\left[\frac{T}{4} - \frac{1}{2w}\left(\sin(wt) - \sin(\frac{wT}{2})\right)\right]$$

$$U^2 = U_m^2\left[\frac{1}{4} - \frac{1}{4p}\sin(p)\right] \Leftrightarrow U^2 = \frac{U_m^2}{4} \Leftrightarrow U = \frac{U_m}{2}$$

The mean value is:

$$U_{mean} = \frac{2}{T}\int_{T/4}^{T/2} U_m \sin(wt)dt$$

$$U_{mean} = \frac{2U_m}{T}\left[-\frac{1}{w}\left(\cos(\frac{wT}{2}) - \cos(\frac{wT}{4})\right)\right]$$

$$U_{mean} = \frac{U_m}{p}$$

The "conventional" multimeter will indicate:

$$U_{volt} = \frac{p}{2\sqrt{2}}U_{mean} \Rightarrow U_{volt} = \frac{1}{2\sqrt{2}}U_m$$

The relationship between the value indicated by a mean-based instrument and a TRMS instrument will be:

$$\frac{U_{volt}}{U}=\frac{\frac{1}{2\sqrt{2}}.U_m}{\frac{1}{2}.U_m}\Leftrightarrow\frac{U_{volt}}{U}=\frac{1}{\sqrt{2}}$$

$$\frac{U_{volt}}{U}=0.707\Leftrightarrow U_{volt}=70{,}7\%U$$

Resulting in an error of 29,3%.

As can be seen from the previous two examples, the error increases with the triggering angle. That may be considered as logic, in empirical terms, as a signal with a very late triggering is quite different from the original sinusoid.

After these particular examples, we can develop a general equation, valid for all triggering angles:

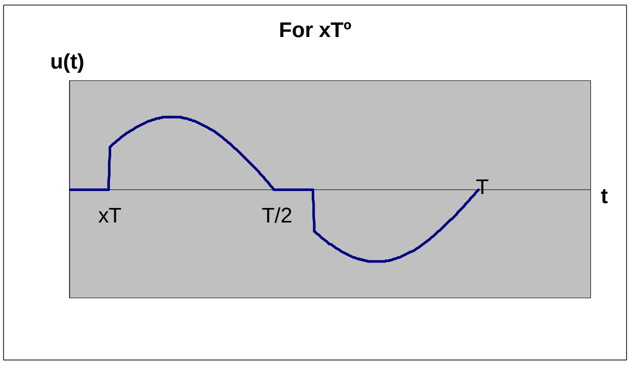

Figure 18 Sinusoidal wave triggered at xT

The RMS expression is the same as before

$$U^2=\frac{2}{T}\int_{xT}^{T/2}U_m^2.\sin^2(wt).dt$$

Considering,

$$\sin\ (wt)=\frac{1}{2}(1+\cos(2wt))$$

We get,

$$U^2=\frac{2.U_m^2}{T}\int_{xT}^{T/2}\frac{1}{2}.(\ 1-\cos(2wt)\ ).dt$$

$$U^2=\frac{U_m^2}{T}\left(\int_{xT}^{T/2}1.dt-\int_{xT}^{T/2}\cos(2wt).dt\right)$$

$$U^2=\frac{U_m^2}{T}\left[t-\frac{1}{2w}.\sin(2wt)\right]_{xT}^{T/2}$$

Since,

$$w=2p/T$$

then,

$$U^2 = \frac{U_m^2}{T}\left[T.\left(\frac{1}{2} - x\right)\left(\frac{T}{4p}\sin(2p) - \sin(4px)\right)\right]$$

$$U^2 = U_m^2.\left(\frac{1}{2} - x + \frac{1}{4p}.\sin(4px)\right)$$

$$U = U_m.\sqrt{\frac{1}{2} - x + \frac{1}{4p}.\sin(4px)}$$

The mean value may be determined:

$$U_{mean} = \frac{2}{T}\int_{xT}^{T/2} U_m.\sin(wt).dt$$

$$U_{mean} = \frac{2.U_m}{T}\left(-\frac{1}{w}\left(\cos(\frac{2w}{T}) - \cos(wxT)\right)\right)$$

$$U_{mean} = \frac{U_m}{p}(1 + \cos(2px))$$

What the mean-based multimeter indicates is:

$$U_{volt} = \frac{p}{2\sqrt{2}}.U_{mean} \Rightarrow U_{volt} = \frac{1}{2\sqrt{2}}.(1 + \cos(2px)\ ).U_m$$

Resulting in the following formula for the determination of the committed error:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}.(\ 1 + \cos(2px)\ ).\frac{1}{\sqrt{\frac{1}{2} - x + \frac{1}{4p}.\sin(4px)}}$$

Eight cases, including the previous two cases (30º and 90º), are considered next, using the general formula.

For T/24:

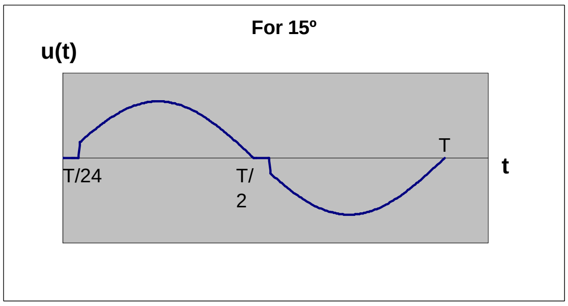

Figure 19 Sinusoidal wave triggered at T/24

The error is:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left(1+\cos(\frac{p}{12})\right)\frac{1}{\sqrt{\frac{11}{24}+\frac{1}{4p}\sin(\frac{p}{6})}}$$

$$U_{volt} = 98.4\% U$$

$$Error : 1.6\%$$

For T/12:

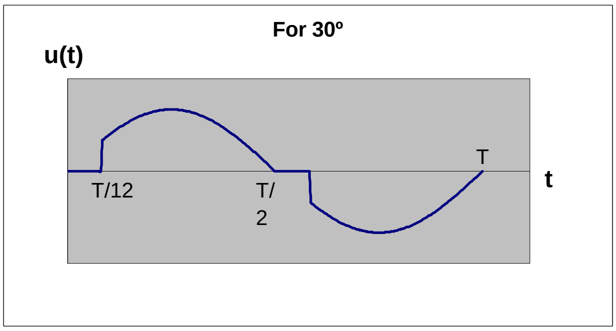

Figure 20 Sinusoidal wave triggered at T/12

The error is:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left(1+\cos(\frac{p}{6})\right)\frac{1}{\sqrt{\frac{5}{12}+\frac{1}{4p}\sin(\frac{p}{3})}}$$

$$U_{volt} = 94.6\% U$$

$$Error : 5.4\%$$

For T/8:

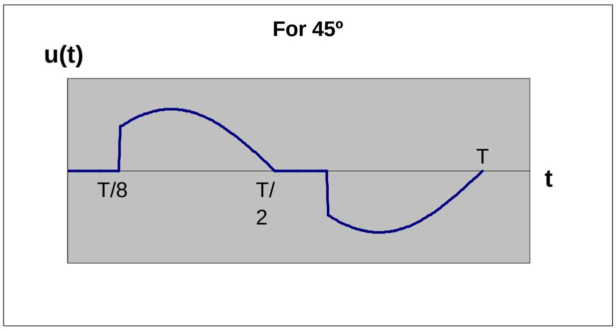

Figure 21 Sinusoidal wave triggered at T/8

The error is:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left(1+\cos(\frac{p}{4})\right)\frac{1}{\sqrt{\frac{3}{8}+\frac{1}{4p}\sin(\frac{p}{2})}}$$

$$U_{volt} = 89.5\% U$$

$$Error : 10.5\%$$

For T/6:

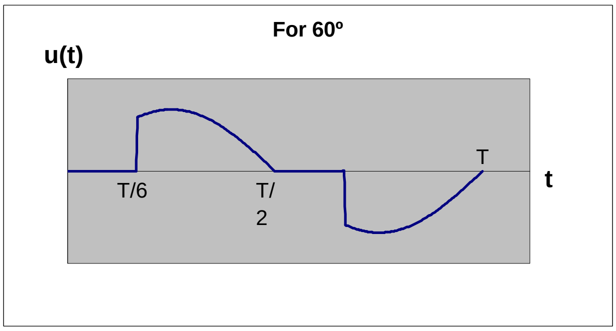

Figure 22 Sinusoidal wave triggered at T/6

The error is:

$$\frac{U_{volt}}{U}=\frac{1}{2\sqrt{2}}\left(1+\cos(\frac{p}{3})\right)\frac{1}{\sqrt{\frac{1}{3}+\frac{1}{4p}\sin(\frac{p}{2})}}$$

$$U_{volt}=83.5\%U$$

$$Error:16.5\%$$

For 5T/24:

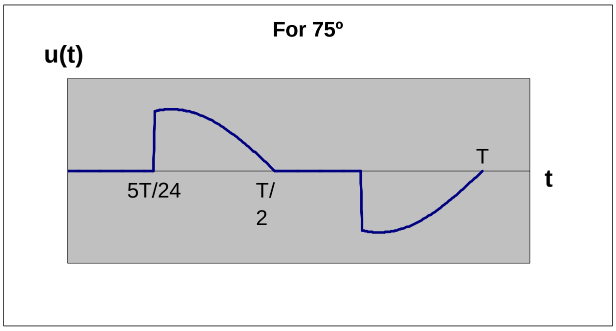

Figure 23 Sinusoidal wave triggered at 5T/24

The error is:

$$\frac{U_{volt}}{U}=\frac{1}{2\sqrt{2}}\left(1+\cos(\frac{5p}{12})\right)\frac{1}{\sqrt{\frac{7}{24}+\frac{1}{4p}\sin(\frac{5p}{6})}}$$

$$U_{volt}=77.3\%U$$

$$Error:22.7\%$$

For T/4:

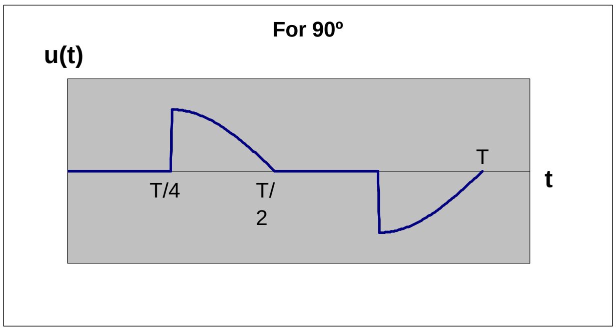

Figure 24 Sinusoidal wave triggered at T/6

The error is:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left(1+\cos(\frac{p}{2})\right)\frac{1}{\sqrt{\frac{1}{4}+\frac{1}{4p}\sin(p)}}$$

$$U_{volt} = 70.7\%U$$

$$Error: 29.3\%$$

For T/3:

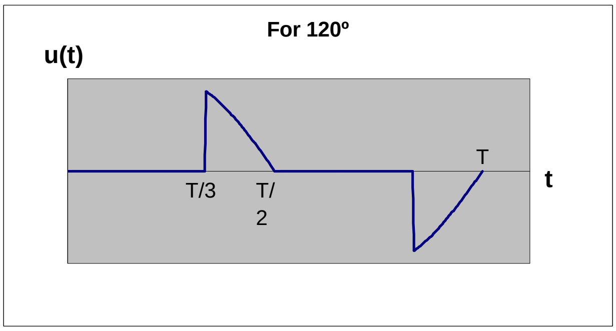

Figure 25 Sinusoidal wave triggered at T/3

The error is:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left(1+\cos(\frac{2p}{3})\right)\frac{1}{\sqrt{\frac{1}{6}+\frac{1}{4p}\sin(\frac{4p}{3})}}$$

$$U_{volt} = 56.4\%U$$

$$Error: 43.6\%$$

For 3T/8:

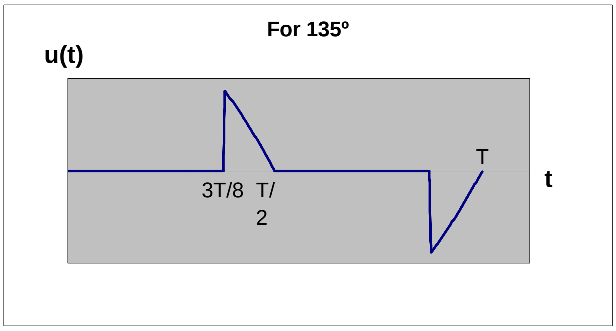

Figure 26 Sinusoidal wave triggered at 3T/8

The error is:

$$\frac{U_{volt}}{U} = \frac{1}{2\sqrt{2}}\left(1+\cos\left(\frac{3p}{4}\right)\right)\frac{1}{\sqrt{\frac{1}{8}+\frac{1}{4p}\sin\left(\frac{3p}{2}\right)}}$$

$$U_{volt} = 48.5\%U$$

$$Error: 51.5\%$$

As it can be seen, the two previously determined errors (for 30º and 90º) are confirmed by the general formula. Moreover, we have proved that the error increases with the triggering angle, becoming an unbearable burden even for rough (low accuracy) measurements.

3.2. "Square" Wave

Pure Alternated

For a "pure" (no DC offset, no duty-cycle) square wave:

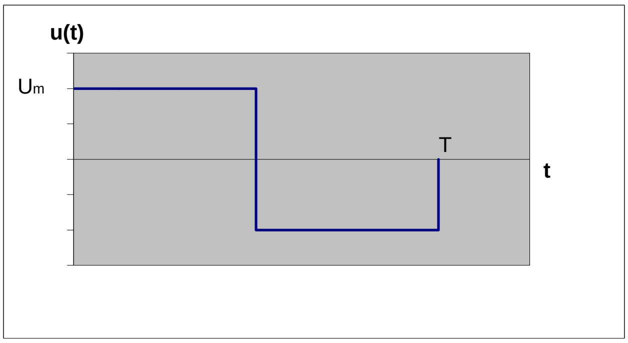

Figure 27 Pure square wave

The RMS value can be determined by the mathematical definition:

$$U^2 = \frac{1}{T}\int_0^T U_m^2 dt \Leftrightarrow U^2 = \frac{U_m^2}{T}\int_0^T dt \Leftrightarrow U^2 = \frac{U_m^2}{T}.T$$

$$U^2 = U_m^2 \Leftrightarrow U = U_m$$

Considering the rectified signal, we can state that:

$U = U_{mean}$

Resulting in a “conventional” multimeter to measure:

$U_{volt}=1{,}11*U_{med}$ or $U_{volt}=111\%*U_{med}$

With an average-responding multimeter, we will get a value 11% higher than the true RMS.

Variable DC Component

If we add a DC component to the square wave:

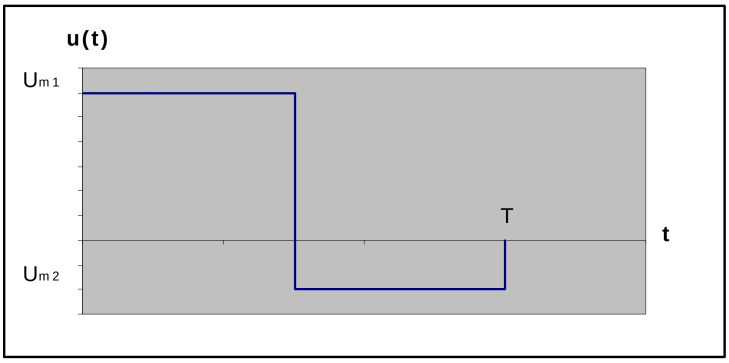

Figure 28 Square wave with DC component

The RMS value can be determined by the mathematical definition:

$$U^2 = \frac{1}{T}\left[\int_0^{T/2} U_{m1}^2 dt + \int_0^{T/2} U_{m2}^2 dt\right] \Leftrightarrow U^2 = \frac{1}{T}\left[U_{m1}^2 + U_{m2}^2\right]\int_0^{T/2} dt \Leftrightarrow U^2 = \frac{1}{2}\left(U_{m1}^2 + U_{m2}^2\right)$$

$$U = \sqrt{\left(U_{m1}^2 + U_{m2}^2\right)\frac{1}{2}}$$

Considering the rectified signal, we can state that:

$U_{mean}=1/2.(U_{m1}+U_{m2})$

The “conventional” multimeter will measure:

$$U_{volt} = \frac{p}{2\sqrt{2}}.U_{mean} \Rightarrow U_{volt} = \frac{1}{4\sqrt{2}}.\left(U_{m1} + U_{m2}\right)$$

The error will be

$$\frac{U_{volt}}{U} = \frac{\frac{1}{4\sqrt{2}}\left(U_{m1} + U_{m2}\right)}{\sqrt{\frac{1}{2}\left(U_{m1}^2 + U_{m2}^2\right)}}$$

Some application examples of this formula are included in ‘6.3. Square Wave’.

Variable Duty-Cycle

A square wave can be generalised to a rectangular wave. That is the same as to vary the signal duty-cycle – the ratio of maximum time to period (total time). For a square wave, the duty-cycle is 50%. So, for a variable duty-cycle square wave:

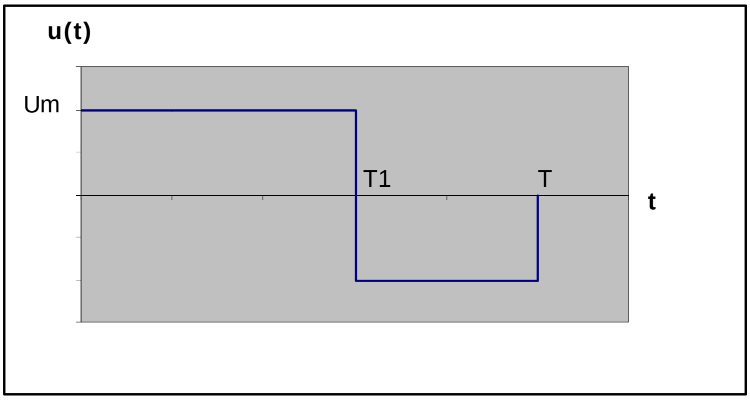

Figure 29 Square wave with variable duty-cycle

The RMS is:

$$U^2 = \frac{1}{T}\left[\int_0^{T1} U_m^2 dt + \int_{T1}^{T} U_m^2 dt\right] \Leftrightarrow U^2 = \frac{U_m^2}{T}\left[\int_0^{T1} dt + \int_{T1}^{T} dt\right] \Leftrightarrow U^2 = \frac{U_m^2}{T}[T1 + T - T1]$$

$$U^2 = U_m^2 \Leftrightarrow U = U_m$$

Considering the rectified signal, we can state that:

$U = U_{mean}$

The “conventional” multimeter will measure:

$U_{volt} = 1{,}11 * U_{mean}$ or $U_{volt} = 111\% * U_{mean}$

The measured signal is again 11% bigger than the TRMS. In fact, we can easily observe that is the same situation as the pure square wave.

Variable DC Component and Duty-Cycle

If we vary both DC offset and duty-cycle:

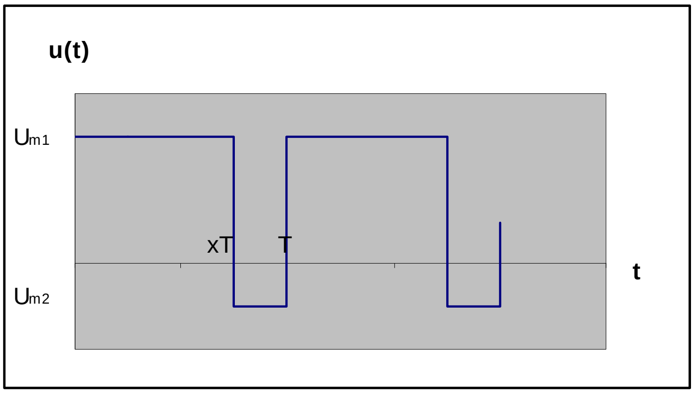

Figure 30 Square wave with variable DC component and duty-cycle

The RMS value can be determined by the mathematical definition:

$$U^2 = \frac{1}{T}\left[\int_0^{xT} U_{m1}^2 dt + \int_{xT}^{T} U_{m2}^2 dt\right] \Leftrightarrow U^2 = \frac{1}{T}\left[U_{m1}^2 \int_0^{xT} dt + U_{m2}^2 \int_{xT}^{T} dt\ \right]$$

$$U^2 = \frac{1}{T}\left(U_{m1}^2[xT] + U_{m2}^2[T - xT]\right)$$

$$U^2 = \left(U_{m1}^2 - U_{m2}^2\right)x + U_{m2}^2$$

$$U = \sqrt{\left(U_{m1}^2 - U_{m2}^2\right)x + U_{m2}^2}$$

Considering the rectified signal, we can state that:

$$U_{mean} = \frac{1}{T}\left[\int_0^{xT} U_{m1} + \int_{xT}^{T} U_{m2}\right]$$

$$U_{mean} = \frac{1}{T}\left(U_{m1}[xT] + U_{m2}[T - xT]\right)$$

$$U_{mean} = \left(U_{m1} - U_{m2}\right)x + U_{m2}$$

The "conventional" multimeter will measure:

$$U_{volt} = \frac{p}{2\sqrt{2}}.U_{mean} \Rightarrow U_{volt} = \frac{p}{2\sqrt{2}}.\left[\left(U_{m1} - U_{m2}\right)x + U_{m2}\right]$$

The general formula for the committed error will then be:

$$\frac{U_{volt}}{U} = \frac{\frac{p}{2\sqrt{2}}\left[\left(U_{m1} - U_{m2}\right)x + U_{m2}\right]}{\sqrt{\left(U_{m1}^2 - U_{m2}^2\right)x + U_{m2}^2}}$$

Some application examples of this formula are included in '6.3. Square Wave'.

3.3. "Triangular" Wave

Pure Alternated

A "pure" triangular wave has the following aspect:

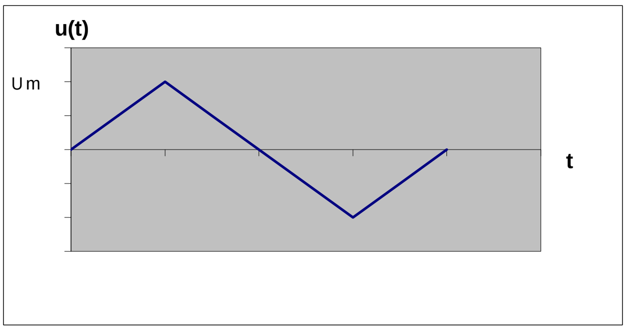

Figure 31 Pure triangular wave

For the sake of simplification, if we consider just a quarter of a period in order to determine the RMS:

$$U^2 = \frac{4}{T}\int_0^{T/4}\left(\frac{4}{T}U_m.t\right)^2 dt \Leftrightarrow U^2 = \frac{64}{T^3}.U_m^2\int_0^{T/4} t^2 dt$$

$$U^2 = \frac{64}{T^3}U_m^2\left[\frac{t^3}{3}\right]_0^{T/4} \Leftrightarrow U^2 = \frac{64}{T^3}U_m^2\,\frac{T^3/64}{3}$$

$$U^2 = \frac{U_m^2}{3} \Leftrightarrow U = \frac{U_m}{\sqrt{3}} \Leftrightarrow U \approx 0{,}58\times U_m \Leftrightarrow U \approx 58\% U_m$$

But the “conventional” multimeter will indicate:

$$U_{mean} = \frac{U_m}{2}$$

$$U_{volt} = \frac{p}{2\sqrt{2}}U_{mean} \Leftrightarrow U_{volt} = \frac{p}{2\sqrt{2}}.\frac{U_m}{2} \Leftrightarrow U_{volt} = \frac{p}{4\sqrt{2}}.U_m$$

$$U_{volt} \approx 0.56\times U_m \Leftrightarrow U_{volt} \approx 56\%\times U_m$$

It is now easy to determine the relationship between the latter and the TRMS:

$$\frac{U_{volt}}{U} = \frac{\frac{p}{2\sqrt{2}}.U_m}{\frac{1}{\sqrt{3}}.U_m} \Leftrightarrow \frac{U_{volt}}{U} = \frac{p\sqrt{3}}{4\sqrt{2}}$$

$$\frac{U_{volt}}{U} \approx 0.96 \Leftrightarrow U_{volt} = 96\% U$$

Resulting in an error of 4%. This value is smaller than in the square wave due to the bigger similarity between the triangular wave and the sinusoidal wave (similar form factors).

Variable DC Component

If we add a constant value to the triangular wave, we get the following waveform:

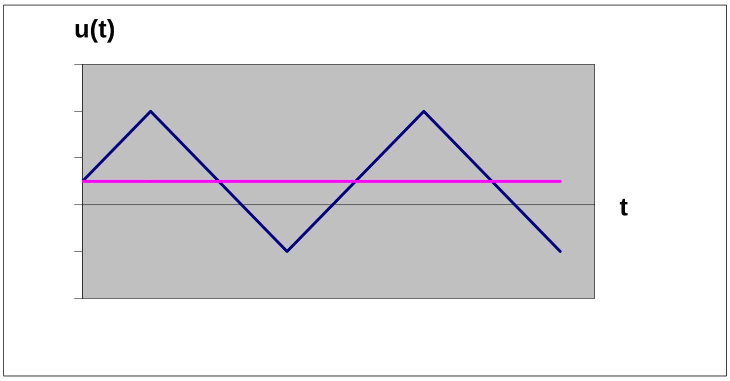

Figure 32 Triangular wave with DC component

The RMS can be determined:

$$U^2 = \frac{1}{T}\int_0^T u^2(t)dt \Leftrightarrow U^2 = \frac{1}{T}\int_0^T \left(U_{DC} + u_{AC}(t)\right)^2 dt \Leftrightarrow$$

$$U^2 = \frac{1}{T}\int_0^T \left(U_{DC}^2 + 2.U_{DC}.u_{AC}(t) + u_{AC}^2(t)\right)dt$$

$$U_{AC}^2 = \frac{4}{T}\int_0^{T/4}\left(\frac{4}{T}U_m.t\right)^2 dt \Leftrightarrow U_{AC} = \frac{U_m}{\sqrt{3}}$$

$$U^2 = \frac{U_{DC}^2}{T}\int_0^T dt + \frac{1}{T}\int_0^T 2.U_{DC}.u_{AC}(t)dt + \frac{1}{T}\int_0^T u_{AC}^2(t)dt$$

$$\frac{2.U_{DC}(t)}{T}\int_0^T u_{AC}(t)dt = 0$$

$$U^2 = U_{DC}^2 + U_{AC}^2 \Leftrightarrow U^2 = U_{DC}^2 + \frac{U_m^2}{3}$$

This result will be generalised for any periodic signal in ‘5.1. RMS DC and AC Components’.

4. (UN)TRUE RMS INSTRUMENTS WORKING PRINCIPLE

Being a “conventional” or TRMS measuring instrument depends on the instrument working principle. This section introduces the most common working principles used in both electromechanical (usually known as “analogue”) and pure electronic instruments (usually known as digital). Only a basic description is undertaken since a deeper study would not be in the scope of this work.

4.1. Moving Coil ⇒ “conventional” instrument

Since 1881, when Jacques d’Arsonval patented the moving coil galvanometer ([Jones, 1991]), that the “d’Arsonval meter movement “ or “permanent magnet/moving-coil (PMMC) meter movement” has been used in different kinds of measuring instruments (ammeters, voltmeters, ohmmeters, etc.):

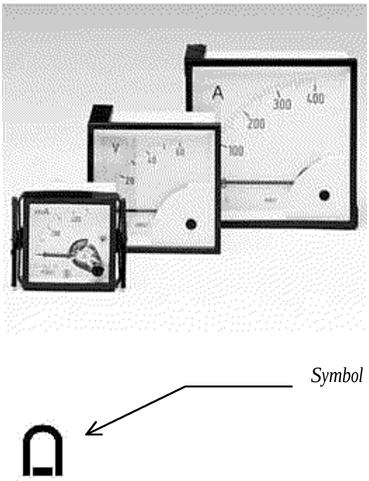

Figure 33: Moving Coil meters and symbol ([Hobut, 1998])

Basically, the interaction between two magnetic fields forces the movement of an indicator. A permanent magnet in the stator generates a constant magnetic field and a coil (in the rotor) generates a magnetic field proportional to the passing current. The rotor will rotate until the point when the magnetic force equals to the force of a spring:

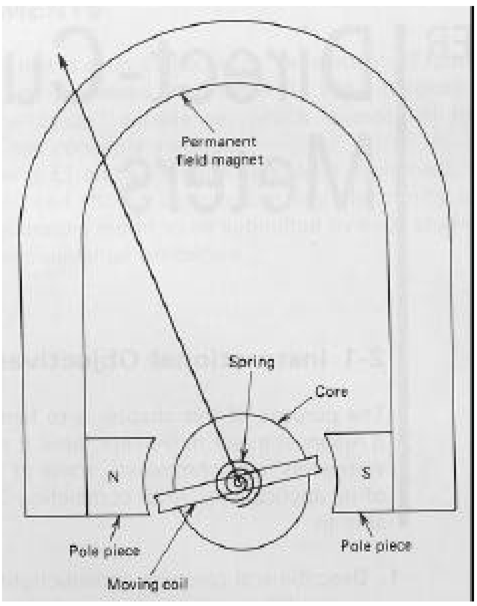

Figure 34: The dÁrsonval meter movement ([Jones, 1991])

It can be proved ([Jacobs, 1968]) that **the movement is directly proportional to the current**:

$$d \propto I$$

Where

d – deviation

I – current

So, for a rectified sinusoid, it will **respond to the signal average**, resulting in a **non-TRMS instrument**.

4.2. Moving Iron ⇒ almost TRMS instrument, but…

A current passing in a coil attracts an iron core, moving a pointer:

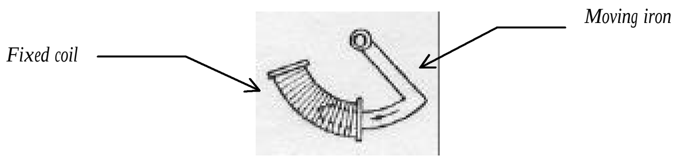

Figure 35: Moving Iron meters and symbol [Jacobs, 1968])

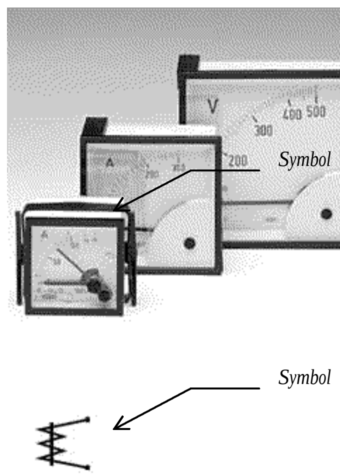

Figure 36: Moving Iron meters and symbol ([Hobut, 1998])

In this type of instruments, it can be proved ([Jacobs, 1968]) that **the movement is directly proportional to the square of the current**:

$$d \propto I^2$$

Theoretically, a measuring instrument with a moving iron working principle is a TRMS meter. The problem is the **limitation in bandwidth**. Due to its inductive behaviour, it is usually restricted to power frequency applications (15-100 Hz). Even for these applications, if the signal has harmonics that are beyond the meter bandwidth, it will not make a correct measurement.

4.3. Electrodynamometer ⇒ almost TRMS instrument, but…

In this kind of measuring instruments, both the stator and the rotor have coils. The interaction between the two magnetic fields results in the movement of a pointer. The most common measuring instruments based on this principle are voltmeters, ammeters and wattmeters:

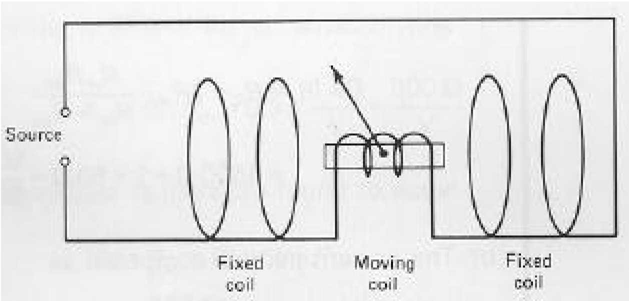

Figure 37: Electrodynamic meter movement ([Jones, 1991])

As in the previous case, it can be proved ([Jacobs, 1968]) that **the movement is directly proportional to the square of the current**. So, in spite of, theoretically, a measuring instrument with a moving iron working principle being a TRMS meter, the **limitation in bandwidth** restricts their use as TRMS instruments.

4.4. Electronic Amplification ⇒ "conventional" instrument

Many current use multimeters include a discrete or integrated amplifier ([Bouwens, 1987, [Helfrick, 1994], [Jones, 1991]), either with an analogue or digital readout (indication). The major part of these measuring instruments determines the RMS based either on the mean value or on the peak value:

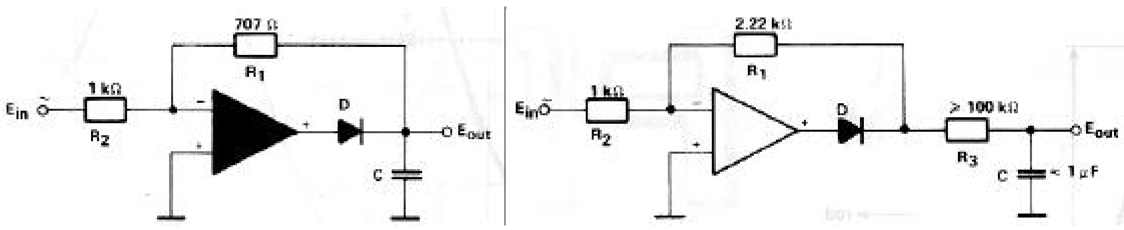

Figure 38: Peak (left) and mean (right) detectors ([Bouwens, 1987)

As such, they only give a correct result for sinusoidal signals and **can not be used as TRMS multimeters**.

4.5. Thermal Effect ⇒ TRMS instrument

The major part of TRMS multimeters are based in the thermal effect working principle, either being electromechanical or pure electronic. This working principle is based in the own physical definition of RMS, i.e., the thermal effect in AC being the same as in DC:

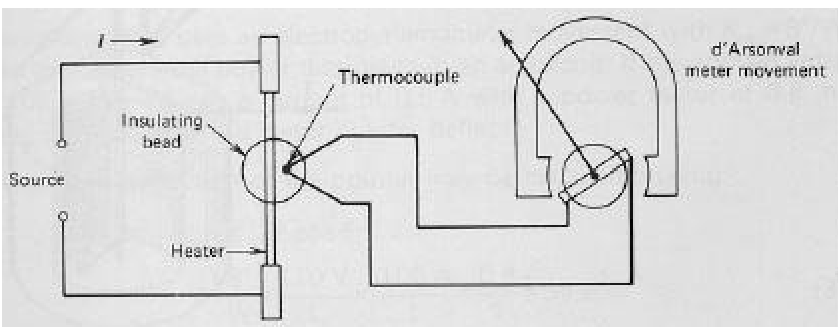

Figure 39: RMS measurement using thermal effect and electromechanical moving-coil indicator ([Jones, 1991])

As may be observed in the previous figure, a voltage or current (the one that must be determined) is applied to a resistance that heats proportionally (to the voltage or current). By means of a temperature transducer, usually a thermocouple, a constant f.e.m. is obtained that is proportional to the square of the input signal ([Fluke, 1994]). This way, a thermal effect instrument **permits the determination of the signal TRMS**. **Bandwidth is usually not a problem** since this kind of principle can be used accurately beyond 50 MHz ([Jones, 1991]).

A more detailed analysis of thermal effect meters is undertaken in [Fluke, 1994].

4.6. Sampling + Math Calculus ⇒ TRMS instrument

All digital-sampling measuring instruments (some multimeters and some oscilloscopes), the RMS value is determined by mathematical calculus. Since the RMS mathematical definition considers a continuous signal, a new expression must be found for a discrete sequence of points (samples). This uses a sum instead of an integral:

$$U^2 = \frac{1}{N}.\sum_{i=1}^{N} s_i^2 \Rightarrow U = \sqrt{\frac{1}{N}.\sum_{i=1}^{N} s_i^2}$$

where

N - number of samples

s_i – value of sample i

We can test this formula very easily. For instance, for a sinusoidal signal with the following general equation:

u(t) = 1. sin (2.π.t)

If we consider 314 (mathematically generated) samples, we will get the following values (using MS Excel, in this example):

N	$\sum_{i=1}^{N} s_i^2$	$U_{mean} = \frac{1}{N}.\sum_{i=1}^{N} \lvert s_i \rvert$	$U = \sqrt{\frac{1}{N}.\sum_{i=1}^{N} s_i^2}$
314	157,1	0,6369	0,7073

Considering the values determined from the continuous equation, we confirm that they are almost equal to the worksheet calculated:

U_{mean} = 0,6366 V U = 0,7071 V

5. THE BANDWIDTH PROBLEM

Since every measuring instrument has a limited bandwidth, and every non-theoretical signal has an infinite bandwidth, the RMS value that is shown does not contain the power of every harmonic. In this section, the (measuring instruments) supplier bandwidth specifications and experimental bandwidth analysis are compared.

5.1. RMS DC and AC Components

Every signal can be decomposed in a DC component (constant value) and an AC component (zero mean). For instance, the following signal has a DC component and a sine wave AC component:

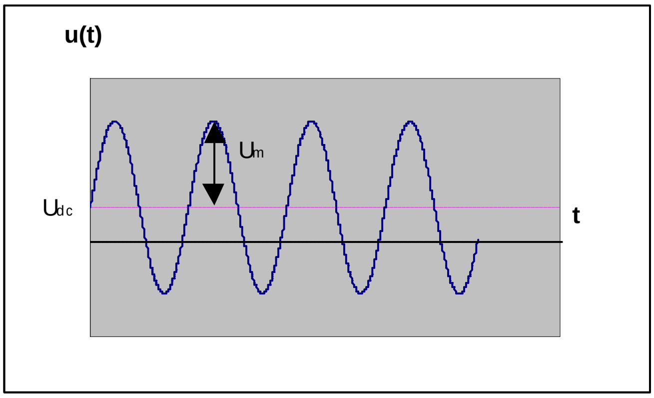

Figure 40: Sine wave plus DC component

Many references state that the signal RMS is then:

$$U_{RMS}^2 = U_{DC}^2 + U_{AC}^2$$

In spite of having inherently used this formula in '3.1. "Sine" Wave', we are now going to prove it for the general case. So, considering that every periodic signal can be decomposed in a DC component (constant value) and an AC component (zero mean):

$$u(t) = u_{DC}(t) + u_{AC}(t)$$

Where

$$u_{DC}(t) = \text{constant} = U_{DC}$$

and

$$\frac{1}{T}\int_0^T u_{AC}(t)dt = 0$$

We can say that:

$$U^2 = \frac{1}{T}\int_0^T u^2(t)dt \Leftrightarrow U^2 = \frac{1}{T}\int_0^T (U_{DC} + u_{AC}(t))^2 dt$$

$$U^2 = \frac{1}{T}\int_0^T \left(U_{DC}^2 + 2.U_{DC}.u_{AC}(t) + u_{AC}^2(t)\right)dt$$

$$U^2 = \frac{1}{T}\int_0^T U_{DC}^2 dt + \frac{1}{T}\int_0^T 2.U_{DC}.u_{AC}(t)dt + \frac{1}{T}\int_0^T u_{AC}^2(t)dt$$

$$U^2 = U_{DC}^2 + \frac{2.U_{DC}(t)}{T}\int_0^T u_{AC}(t)dt + U_{AC}^2$$

Since the AC component has a null mean ($\frac{1}{T}\int_0^T u_{AC}(t)dt = 0$),

$$U^2 = U_{DC}^2 + U_{AC}^2 \Leftrightarrow U = \sqrt{U_{DC}^2 + U_{AC}^2}$$

This expression can be used whenever a measuring instrument indicates AC and DC values separately. Moreover, a periodic signal can be decomposed in a sum of sinusoids (Fourier- Series):

$u(t)=U_{DC} + U_{fundamental} + U_{harmonic1} + \ldots + U_{harmonicN}$

The signal RMS can then be generalised as a function of each frequency component RMS:

$$U = \sqrt{U_{DC}^2 + U_{fundamental}^2 + U_{harmonic1}^2 + \ldots + U_{harmonicN}^2}$$

This value is sometimes called the square-root-of-the-sum-of-the-square (RSS) of its components [Fluke, 1994].

Since measuring instruments have a limited bandwidth, they cannot properly integrate the contribution of the energy in all of the harmonics to the total value. A more detailed analysis can be found in [Fluke, 1994].

As a remark, there are many theoretical factors associated to the composition of waveforms and their measurement. As explained in [Fluke, 1994], the primary areas to evaluate when assessing whether an instrument is suitable for measuring a given waveform are its:

- DC response
- Harmonic response
- Internal phase shift
- Bandwidth
- Dynamic amplitude range

All of these factors may influence the way an instrument measures the RMS of a signal. For instance, as we will see in the RMS experimental analysis (‘6. RMS Experimental Analysis’), “conventional” multimeters have a “strange” behaviour when measuring signals with a DC offset. Harmonic Distortion and Internal Phase Shift have to do with the instrument’s ability to “detect” odd and even harmonics with different phase shifts related to the fundamental. The Bandwidth restriction is the subject of this chapter.

The Dynamic Amplitude Range characterises the instrument capability of measuring signals with a certain Crest Factor (U_m/U). For instance, the BK PRECISION TRMS multimeter used in the experiments has a maximum Crest Factor of 3. That means that if the peak value exceeds 3 times the RMS value, the instrument is no longer capable of performing a correct measurement.

5.2. Instrument Bandwidth Specifications

The following table was constructed from the instrument specification manual:

Supplier	Model	Bandwidth
Beckman Industrial	DM25L	not defined
Yokogawa	2013	not defined
BK precision	391	20 Hz - 20KHz
Fluke	11	50 Hz - 400 Hz
Velleman	AVM360	not defined
Metrix	MX1	16 Hz - 1KHz
Fluke	45	20 Hz - 100 KHz
Fluke	123	DC - 20 MHz
Philips		DC - 100 MHz

5.3. Instrument Bandwidth Experimental Analysis

The following table represents the experimental bandwidth results with all the used instruments, from 10 Hz until 2 kHz:

frequency	dm25l	Velleman	fluke11	metrix	Fluke45	test bench	PM3355	Fluke123
10	7.15	6.8	6.92	6.9	6.958	6.902	7.18	7.10
50	7.08	6.8	7.03	6.9	7.037	7.082	7.17	7.08
100	7.08	6.8	7.04	6.9	7.041	7.094	7.17	7.08
200	7.08	6.8	7.04	6.9	7.044	7.100	7.14	7.07
400	7.10	6.85	7.04	6.9	7.047	7.100	7.10	7.07
600	7.12	6.85	7.04	6.9	7.048	7.100	7.08	7.07
800	7.16	6.85	7.05	6.9	7.050	7.100	7.04	7.07
1000	7.21	6.85	7.05	6.9	7.051	7.113	7.02	7.07
1200	7.28	6.85	7.05	6.9	7.051	7.117	7.00	7.07
1400	7.35	6.85	7.05	6.9	7.052	7.120	7.00	7.07
1600	7.43	6.85	7.06	6.9	7.053	7.124	6.99	7.07
1800	7.53	6.85	7.06	6.9	7.054	7.127	6.96	7.07
2000	7.63	6.85	7.07	6.9	7.054	7.131	6.96	7.07

These values are represented in the following graphic:

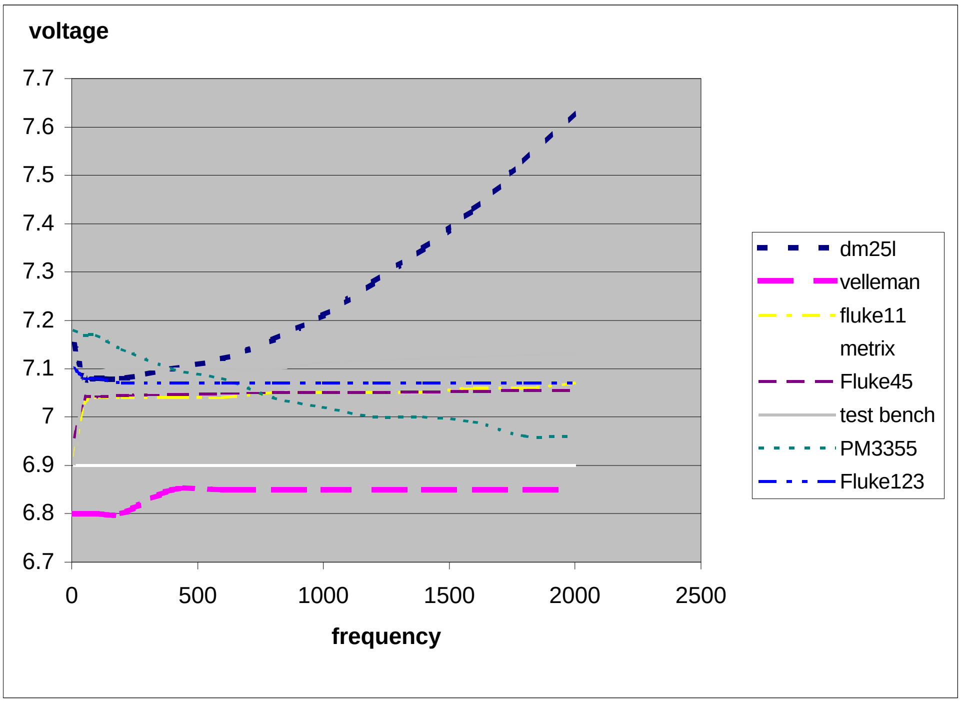

Figure 41: Instrument bandwidth (0-2000 Hz)

Bandwidth experimental analysis was undertaken until 20 kHz, as it is shown next:

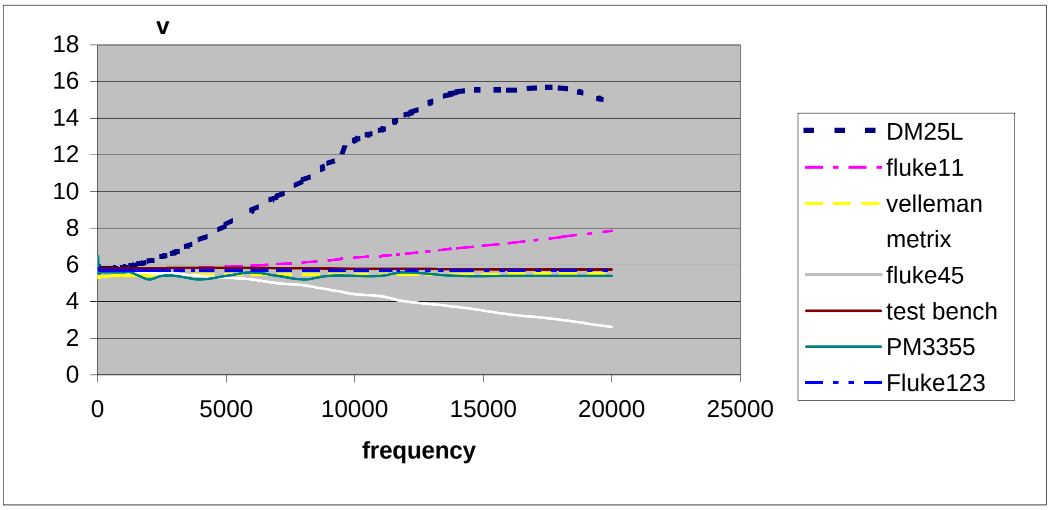

Figure 42: Instrument bandwidth (0-20000 Hz)

Unless DML25, every instrument can measure signals with frequency components until 5 kHz. DML25 seems to be suited until 500-1000 Hz.

6. RMS EXPERIMENTAL ANALYSIS

In this chapter, the RMS of several waveforms is measured, targeting the validation of the analytical analysis undertaken in '3. "Conventional" Multimeters are not good for Non-Sinusoidal Signals'.

6.1. Used Measuring Instruments

The experimental analysis was developed using the following multimeters and oscilloscopes:

Supplier	Model	Kind	TRMS
Beckman Industrial	DM25L	multimeter	No[1]
Fluke	11	multimeter	No[1]
Velleman	AVM360	multimeter	No[1]
Metrix	MX1	multimeter	No[1]
Yokogawa	2013	multimeter	No[2]
Fluke	45	multimeter	Yes
BK Precision	391	multimeter	Yes
Philips	PM3355	oscilloscope	Yes
Fluke	123	oscilloscope	Yes

[1] moving coil instrument.

[2] moving iron instrument.

6.2. Sine Wave

"Pure" Sinusoid

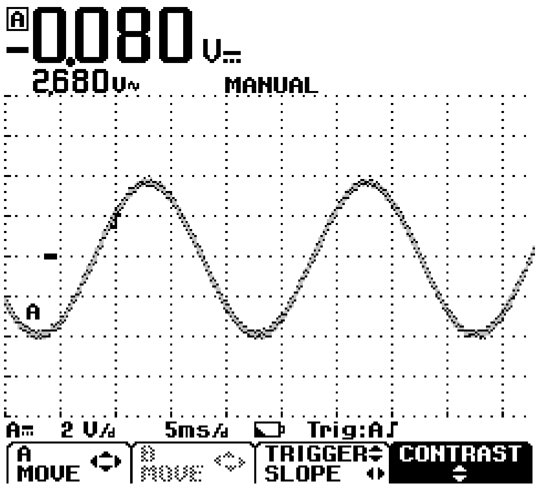

Figure 43: Generated sinusoidal signal

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
2,7	2,67	2,6	2,7	2,68	2,71	2,69	2,68

We can note that conventional multimeters and TRMS instruments indicate the “same” RMS value, corresponding to the theoretical analysis.

The signal spectrum shows that the generated (and measured) signal is not a pure sinusoid though:

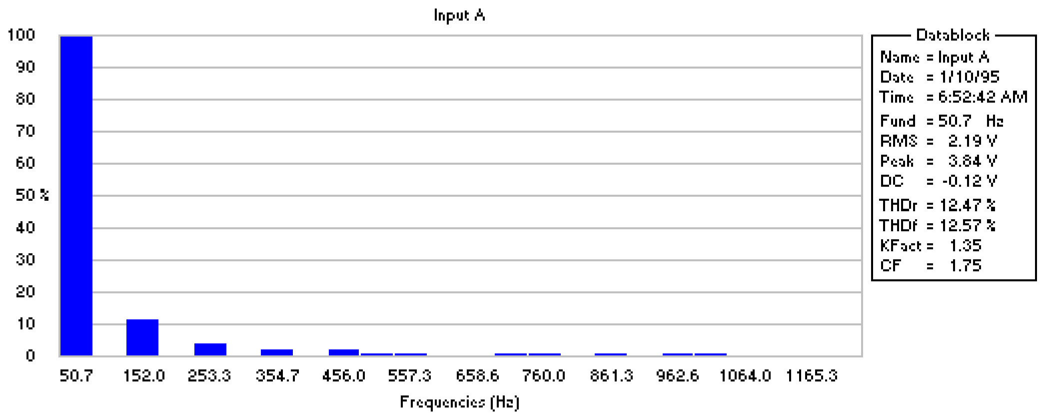

Figure 44: Generated sinusoidal signal spectrum

As can be seen in the signal spectrum, in spite of the generated signal not being truly sinusoidal, every harmonic is inside the instruments bandwidth, resulting in similar RMS values for all instruments.

Variable Triggering angle

The tests for the signals according to were carried out starting from a bulb and of a switch making it possible to manage the power of the signal. However the maximum power brought is that for an angle of 45º and the minimum capacity making it possible to give us a signal on the oscilloscope is of approximately 125º

For 45º

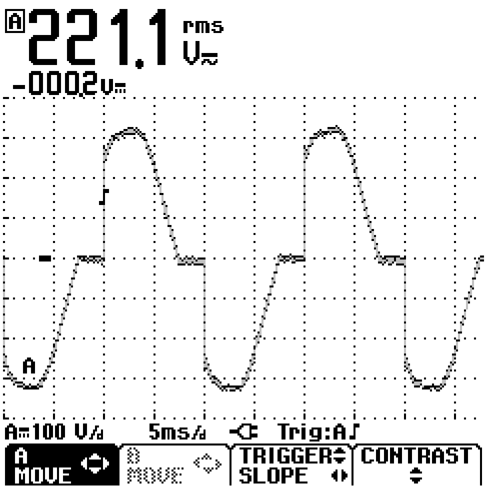

Figure 45: 45º triggered sinusoid signal

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
200	200	200	200	225	223	222	221

Averaging “conventional” and TRMS instrument results separately:

$U_{volt}/U = 200/222.75 = 0.89$

The error is 11%. Theoretically it was 10.5% so the two values correspond.

The signal has the following spectrum:

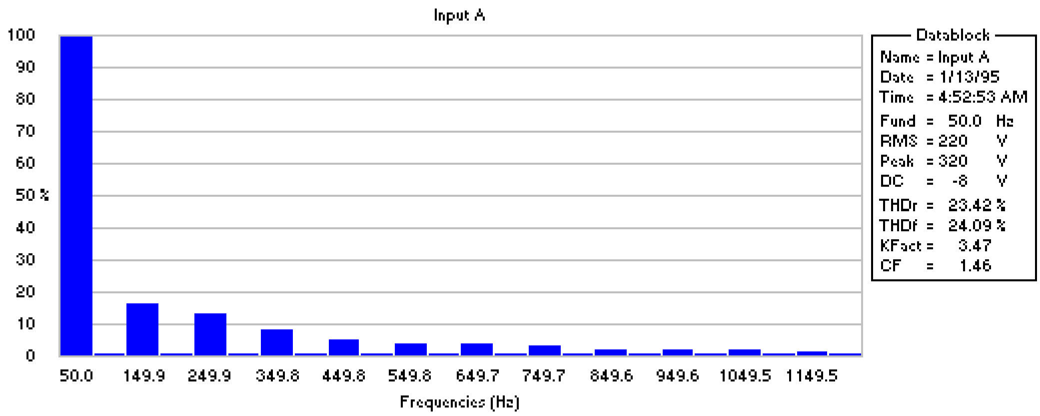

Figure 46: 45º triggered sinusoid signal spectrum

Note stronger (non-fundamental) harmonic power and a crest factor of 1.46.

For 90º:

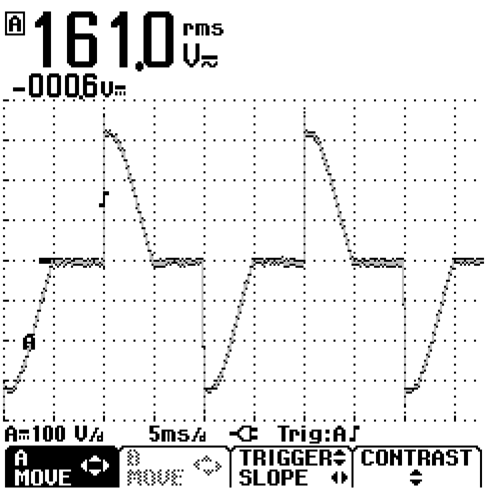

Figure 47: 90º triggered sinusoid signal

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
116	116	115	115	162,26	162,3	161	161

Averaging "conventional" and TRMS instrument results separately:

$$U_{volt}/U = 115.5/161.64 = 0.714$$

The error is 29.6%, it is very close to that calculated (29.3%).

The signal has the following spectrum:

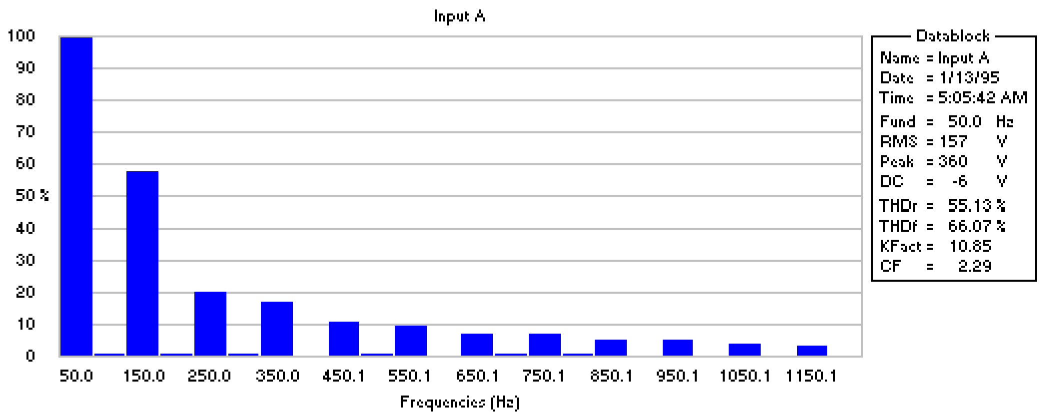

Figure 48: 90º triggered sinusoid signal spectrum

Note that 3th harmonic is more than 50% of fundamental and the crest factor is now 2.29.

For 120º

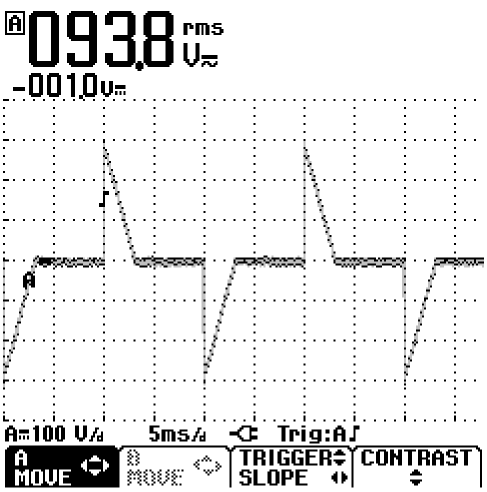

Figure 49: 120º triggered sinusoid signal

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
52	53	55	55	92,74	93,4	93	93,4

Averaging "conventional" and TRMS instrument results separately:

$$U_{volt}/U = 53.75/93.135 = 0.577$$

The experimental error is 42.3%, while the analitically calculated was 43.6%.

Note that in the signal spectrum, harmonic distortion is even bigger and the crest factor is now beyond 3 (3.27):

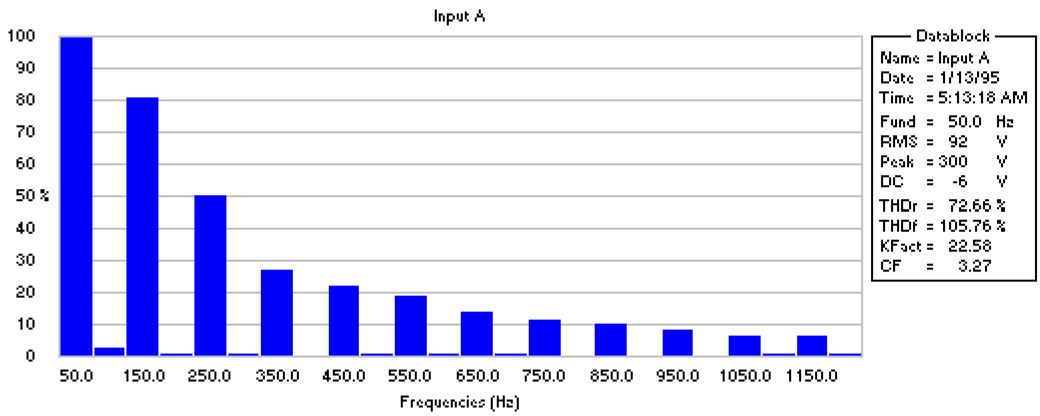

Figure 50: 120º triggered sinusoid signal spectrum

6.3. Square Wave

Pure Alternated

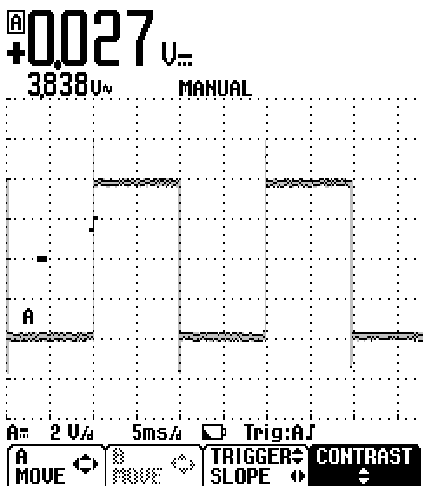

Figure 51: Generated square wave

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
4,29	4,25	4	4,2	3,83	3,87	3,85	3,84

The error obtained between the two types of apparatus will have to be 11% according to the mathematical analysis. Averaging "conventional" and TRMS instrument results separately:

$$U_{volt}/U = 4.185/3.845=1.09$$

One notes an error of more than 9% which is similar to the theoretically determinated.

The generated square wave spectrum is:

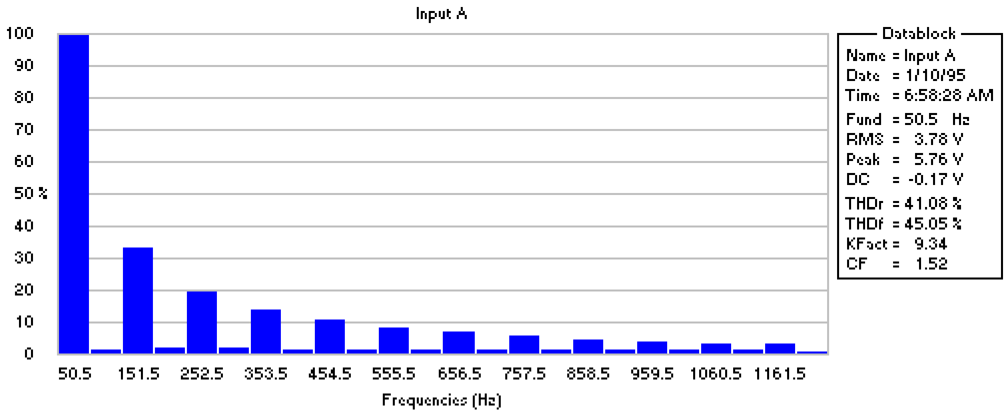

Figure 52: Generated square wave spectrum

As we can see, there is a small portion of power in even harmonics, which may “upset” the measuremnt in some instruments ([Fluke, 1994]).

Variable DC component

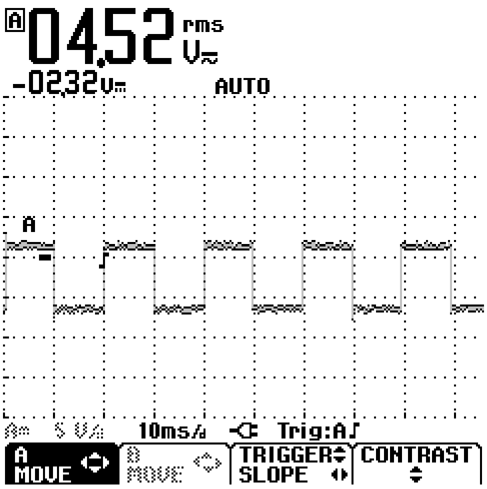

Figure 53: Generated square wave with DC component

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
4.35	2.4	1.5	3.8	4.52	3.9	4.51	4.52

We conclude that the DC component turns the RMS measurement in non TRMS multimeters impossible, as is stated in [Fluke, 1994].

The generated square wave spectrum is much similar to the previous, but with a –2.41 V DC component:

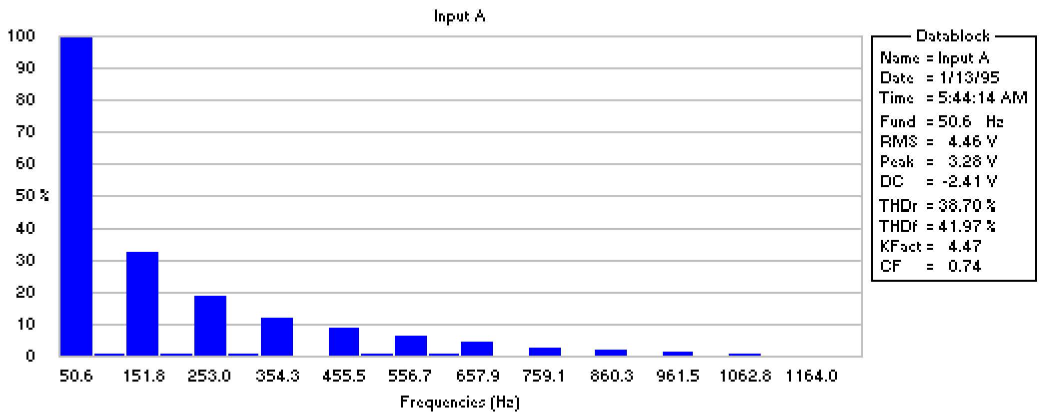

Figure 54: Generated square wave with DC component spectrum

Another example of how “conventional” multimeters get confused with DC components:

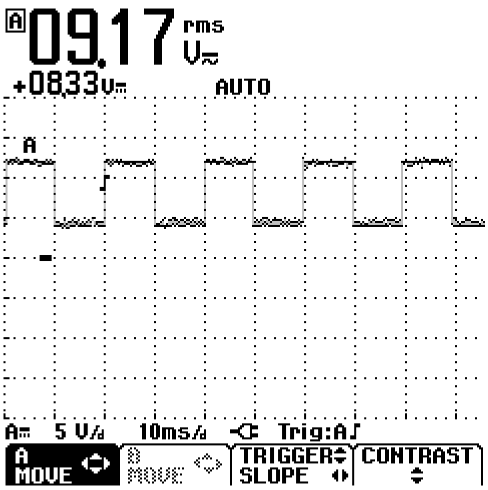

Figure 55: Generated square wave with DC component - 2

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
4.29	8.3	26	10.5	9.15	3.87	9.35	9.17

Again, all the “conventional” multimeters give different values. On the other hand, all TRMS instruments indicate approximate values.

Variable Duty-Cycle - #1

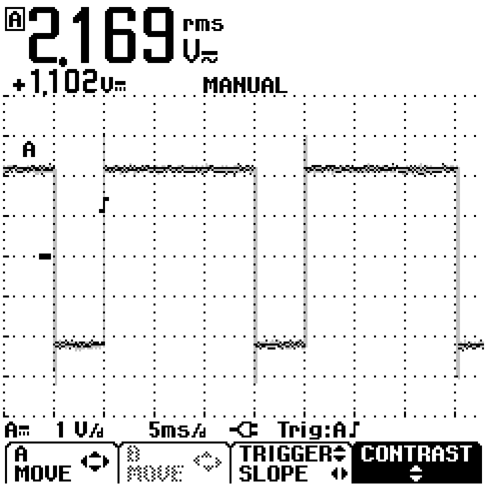

Figure 56: Generated square wave with variable duty-cycle - #1

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
1.8	1.77	3	2.6	2.16	1.869	2.28	2.17

Here, we get the same problem, because the signal will have a DC component (around 1 V) and a very "strage" spectrum, with both significant odd and even harmonics.

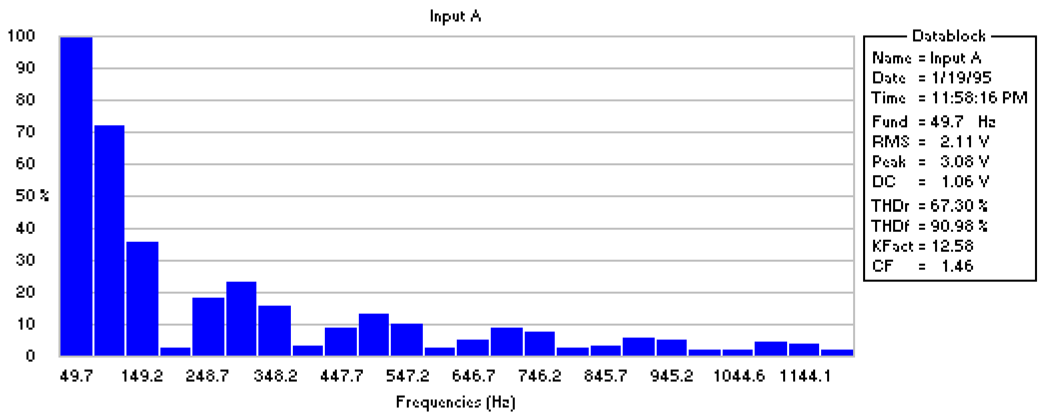

Figure 57: Generated square wave with variable duty-cycle spectrum - #1

Variable Duty-Cycle - #2

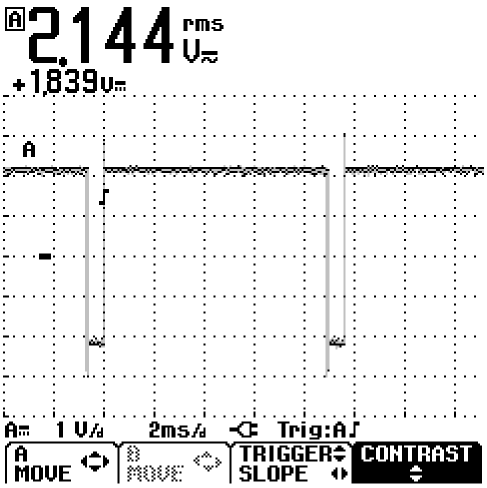

Figure 58: Generated square wave with variable duty-cycle - #2

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
0.65	0.63	3.8	2.7	2.13	1.1	1.1	2.144

Here, the results are even worse and the spectrum is has even more harmonic distortion:

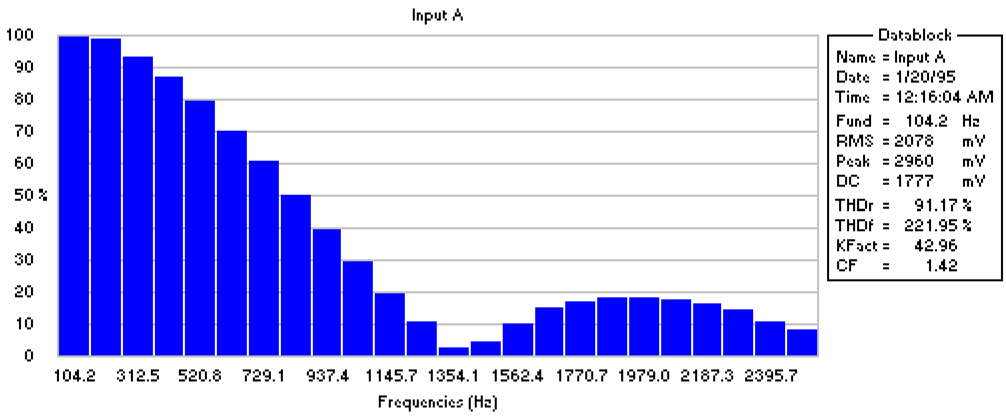

Figure 59: Generated square wave with variable duty-cycle spectrum - #2

Duty Cycle + DC component - #1

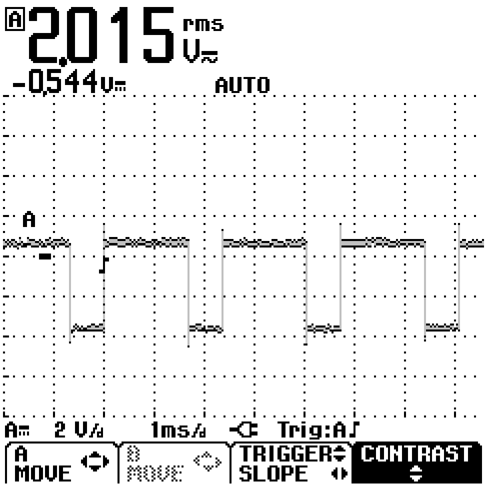

Figure 60: Generated square wave with variable DC component and duty-cycle - #1

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
2.05	1.9	0.7	1.5	2.01	1.95	2	2.01

This waveform is similar to the previous one. The only difference is that now we explicitly add the DC offset with the signal generator. The signal spectrum is:

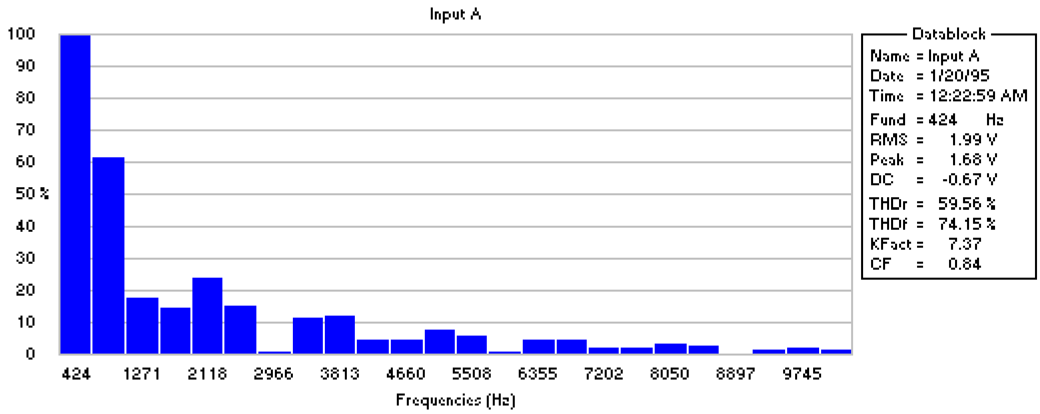

Figure 61: Generated square wave with variable DC component and duty-cycle spectrum - #1

Duty Cycle + DC component - #2

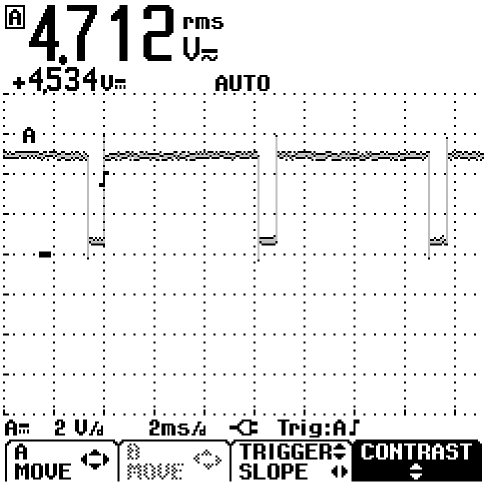

Figure 62: Generated square wave with variable DC component and duty-cycle - #2

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
0.89	0.84	10	5.8	4.7	1.28	4.78	4.71

The values are even more divergent, due to a more scattered spectrum and a more significant DC component (more than 4 V):

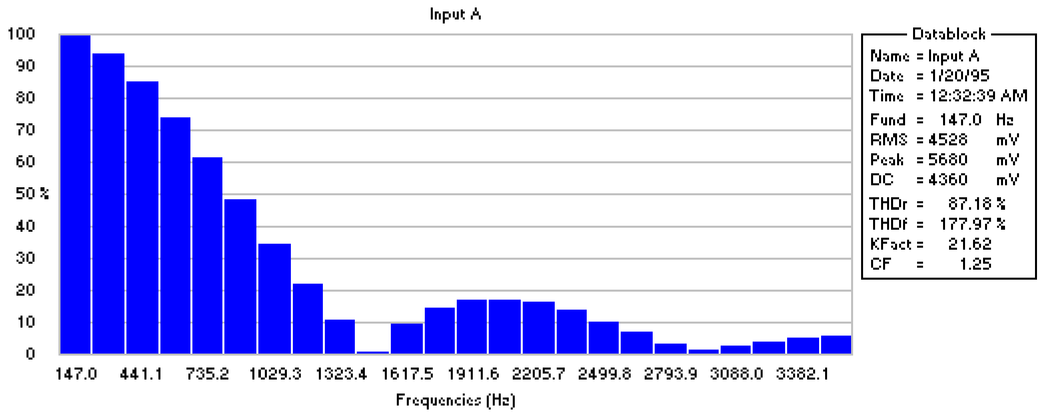

Figure 63: Generated square wave with variable DC component and duty-cycle spectrum - #2

6.4. Triangular Wave

Pure alternated

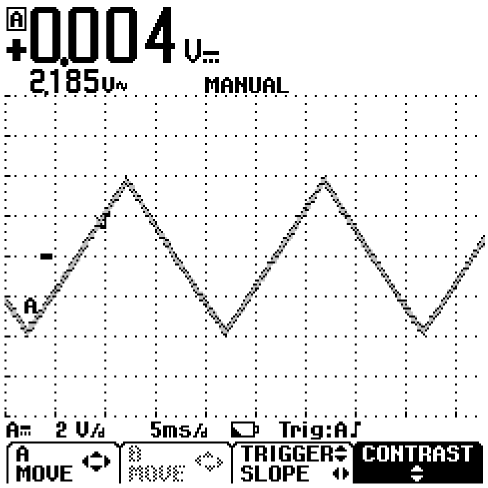

Figure 64: Generated triangular wave

The following RMS values were obtained (in Volts):

DM25L	Fluke11	Velleman	Metrix	Fluke45	Test Bench	PM3355	Fluke123
2,11	2,09	2	2,1	2,181	2,2	2,19	2,185

Averaging “conventional” and TRMS instrument results separately:

$U_{volt}/U = 2.075/2.189 = 0.95$

The experimental result (5%) complies with the theoretical result (4%).

The triangular wave spectrum is represented next:

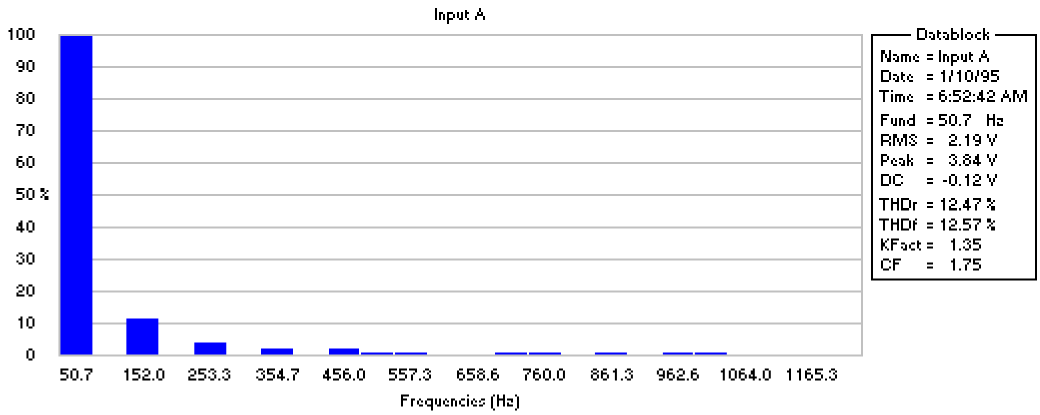

Figure 65: Generated triangular wave spectrum

The (unwanted) DC component (– 0.12 V) may have a small influence in the results. On the other hand, harmonic components are not significant and the crest factor is low (1.75).

6.5. Worksheet Mathematical Calculus

In order to confirm the discrete RMS formula, a digitally sampled sinusoidal signal was used as an example:

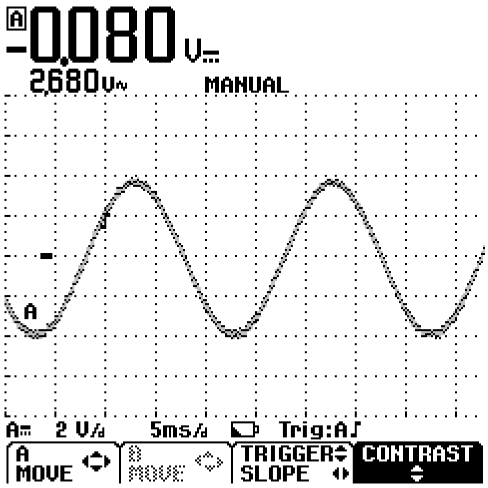

Figure 66: Generated sine wave

As can be seen in the spectrum, the RMS value is 2.69 V, with a DC component of – 0.23 V.

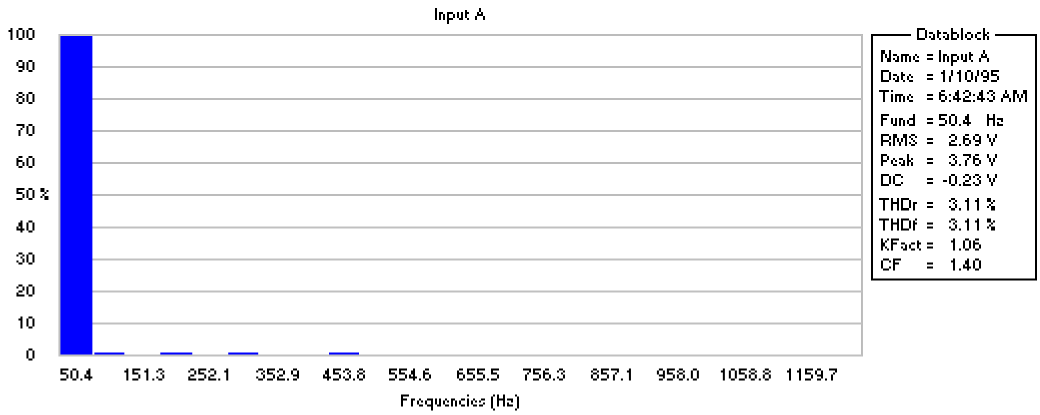

Figure 67: Generated sine wave spectrum

Data values were processed in MSExcel worksheet, giving the following results:

N	$\sum_{i=1}^{N} s_i^2$	$U = \sqrt{\frac{1}{N}.\sum_{i=1}^{N} s_i^2}$
254	1949.325	2.77029

The mathematically processed RMS is 2.77 V, approximately 3% bigger than the Fluke123 indication.

6.6. Analytical vs. Experimental Results

The first important assumption is that all the experimental work was carried out without having the measuring instruments calibrated. In order to have a complete confidence in the experimental results, we should guarantee that the instruments uncertainty specifications were accordant to their “real” accuracy. Unfortunately, there was no possibility of having the instruments calibrated before the beginning of the project. Moreover, uncertainty was not considered when presenting the readouts. If a more deep analysis was carried out, the uncertainty interval for each measurement should be determined. In the aim of this work, we made the assumption that, as all the instruments had an uncertainty less than 2-3%, the indicated values where “very near” the “truthful” value.

According to the various mathematically and experimentally obtained results, we have shown that there is a big similarity between analytical and experimentally obtained errors. The only exception is with respect to the experimental analysis of signals with DC component. This problem is referred in ‘5. The Bandwidth Problem’.

Even the small differences between analytical and experimental analysis can be explained. One of the major causes for this is that analytical analysis was done considering “pure” waveforms, i.e., they correspond to a mathematical expression. On the other hand, the experimental analysis depended on a signal generator. It is quite obvious that this kind of device is not able to generate a “pure” waveform. For instance, a sine wave is easily represented in the analytical analysis, but when we go the practical field, the “real” signal that is generated is not a “pure” sinusoid.

7. OVERVIEW

In this work, an analytical vs. experimental analysis of true and untrue RMS measuring instruments was carried out. Several “conventional” and TRMS multimeters were used, as well as two digital sampling oscilloscopes (TRMS, of course). One of the latter was connected to a PC via an optical/RS-232 interface, where a virtual instrument application permitted the acquisition of all the generated waveforms, spectrum calculation and the insertion of all those waveforms in this document.

As it was already mentioned, for a perfect validation of this work, a guaranty that all the measuring instruments had their uncertainty according to supplier’s specification was necessary. Only the Fluke 123 Oscilloscope was within the calibration period, since it was bought during the development of the project.

The signal generator that was used is very rudimentary. A programmable one would be the best solution for this kind of work. For instance, it did not allow us to carry out the experimental analysis for a triggered sine wave. To overcome this problem, a light regulator (dimmer) using a triac was used. As a remark, it is important to mention that most of the light regulators used nowadays do not have this working principle; they use a auto-transformer.

The experimental analysis was almost completely successful in confirming the analytical calculus. As it was already explained, that was not achieved for the signals with DC component, due to “conventional” multimeter limitations.

Future work in this area could focus on a deeper analytical and experimental analysis taking into account all the factors that can affect a RMS measurement (‘5.1. RMS DC and AC Components’): DC response, harmonic response, internal phase shift, bandwidth and dynamic amplitude range. The use of a programmable signal generator with communication capabilities (RS-232 or GP-IB) would, permit the generation of almost any waveform and also a high degree of automation in measurement, where a PC could control the signal generator and, at the same time, make all the readouts acquisition.

8. REFERENCES

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