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# A novel hybrid fusion algorithm to bridge the period of GPS outages using low-cost INS



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## ABSTRACT

Land Vehicle Navigation (LVN) mostly relies on integrated system consisting of Inertial Navigation System (INS) and Global Positioning System (GPS). The combined system provides continuous and accurate navigation solution when compared to standalone INS or GPS. Different fusion methodology such as those based on Kalman filtering and particle filtering has been proposed that estimates and models the INS error during the GPS signal availability. In the case of outages, the developed model provides an INS error estimates, thereby improving its accuracy. However, these fusion approaches possess several inadequacies related to sensor error model, immunity to noise and computational load. Alternatively, Neural Network (NN) based approaches has been proposed. In the case of low-cost INS, the NN suffers from poor generalization capability due to the presence of high amount of noises.

The paper thus introduces a novel and hybrid fusion methodology utilizing Dempster–Shafer (DS) theory augmented by Support Vector Machines (SVM), known as DS-SVM. The INS and GPS data fusion is carried using DS fusion whereas SVM models the INS error. During GPS availability, DS provides accurate solution; whereas during outages, the trained SVM model corrects the INS error thereby improving the positioning accuracy. The proposed methodology is evaluated against the existing Artificial Neural Network (ANN) and the Random Forest Regression (RFR) methodology. A total of 20–87% improvement in the positional accuracy was found against ANN and RFR.

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## 1. Introduction

In a land vehicle navigation, answers to the fundamental questions such as “What is my current location or Am I heading in the right direction?” can easily be answered using Global Positioning System (GPS) derived navigation parameter. GPS is a satellite-based radio navigation system developed by the United States Department of Defense (DoD) to provide accurate absolute positioning information over extended periods of time worldwide under all weather conditions. Although GPS has been widely used in land vehicle navigation systems, standalone GPS is unable to provide continuous and reliable navigation solutions in the presence of signal fading and/or blockage such as in urban areas. Thus, to bridge the period of GPS outages, Inertial Navigation System (INS) is utilized. INS is a self-contained system that consists of an Inertial Measurement Unit (IMU) and an onboard computer to process the raw IMU measurements. Complete IMU comprises of three

set of accelerometers and gyroscopes placed along the three mutually orthogonal directions capable of measuring vehicle linear accelerations and angular velocity. However, due to the presence of noises in the raw IMU measurements the standalone INS solution drifts with time depending upon the grade of INS. An integrated INS/GPS system combines the advantages of both the techniques by reducing INS errors and continuously provides reliable navigation data. Thus, to reduce the standalone INS drift, its errors are modeled using suitable integration methodology, generally with GPS. The GPS compliments INS in its error estimation process by providing a reference solution. On the other hand, INS bridge GPS signal gaps, assist in signal reacquisition after an outage and reduces the search domain for detecting and correcting GPS cycle slips (El-Rabbany, 2002; Wong, Schwarz, & Cannon, 1988). The combined system thus overcomes the disadvantages of each other, while maintaining the continuity and accuracy of the navigation solution.

The INS and GPS integration process estimates and model the INS error as long as the GPS signals are available and simultaneously delivers the accurate and high rate navigation parameter. Different Bayesian filtering approaches such as the Kalman Filter (KF), Extended Kalman Filter (EKF) and the Particle Filter (PF) have

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been proposed and implemented to integrate the INS and GPS data. The KF is an optimal filter for linear systems with Gaussian noises but is not applicable to non-linear systems (Hosteller & Andreas, 1983; Vanicek & Omerbasic, 1999). For non-linear models, EKF (i.e., linearized KF) can be implemented which is based on linearization of the system and measurement models. However, the linearization process is often complicated and may cause filter divergence (Arulampalan, Maxwell, Gordon, & Clapp, 2002). The PF is suggested and implemented by a number of researchers (Arulampalan et al., 2002; Doucet, Freitas, & Gordon, 2001; Ristic, Arulampalan, & Gordon, 2004). In PF, the posterior distribution is represented by a cluster of random particles rather than a linearized function as in EKF. However, the basic PF may require a large number of particles, making the algorithm computationally expensive (Aggarwal, Syed, & El-Sheimy, 2008; Arulampalan et al., 2002).

Alternatively, Artificial Intelligence (AI) approaches such as Multi-Layer Perceptron Neural Networks (MLPNN), Radial Basis Function Neural Networks (RBFNN) and Adaptive Neuro-Fuzzy Inference System (ANFIS) have gained the popularity in recent years, due to its ability to deal with the problem of non-linearity (El-Sheimy, Chiang, & Noureldin, 2006; El-Sheimy, Chiang, & Noureldin, 2008; Hiliuta, Landry, & Gagnon, 2004; Noureldin, El-Shafie, & Taha, 2007; Noureldin, Osman, & El-Sheimy, 2004; Reda Taha, Noureldin, & El-Sheimy, 2003; Semeniuk & Noureldin, 2006; Sharaf, Noureldin, Osman, & El-Sheimy, 2005; Sharaf, Tarbouchi, El-Shafie, & Noureldin, 2005). El-Sheimy et al. (2006), proposed the Position Update Architecture (PUA), and Position and Velocity Updates Architecture (PVUA) utilizing three layer multi-layer perceptrons (MLP-3) neural network to integrate the INS and GPS data. The basic principle behind these architectures utilizing Artificial Neural Network (ANN) is to mimic the latest vehicle dynamic as long as the GPS signals are available. During the training process ANN is trained to model the input–output functional relationship relating INS and GPS data. In the case of outages, the trained model is utilized to estimate the reliable navigation solution using INS solution as input. Though the ANN based architecture performs better than KF approaches as explained in El-Sheimy et al. (2006), the accuracy of these architectures degrades in case of low-cost INS. This is mainly due to the presence of high inherent INS sensor errors (like turn-on to turn-on biases, in-run biases and scale factor drifts) that increases the non-linear complexity of the input–output functional relationship to be modeled. This limits the ANN generalization ability and thus affects its prediction accuracy. On the other hand Adaptive Neuro Fuzzy Inference System (ANFIS) proposed in Hiliuta et al. (2004), Reda Taha et al. (2003); Sharaf, Noureldin, et al. (2005), Sharaf, Tarbouchi, et al. (2005) possess some limitations regarding the ANFIS parameter optimization which results in huge computation load. As a result its real time implementation is affected.

In this research, we aim at developing a novel and hybrid GPS/INS integration module, based on Dempster Shafer theory and Support Vector Machine (SVM), known as DS-SVM. The DS theory based on the neo-classical idea of mass or belief as opposed to the well-understood probabilities of Bayesian theory (Dempster, 1967; Shafer, 1976; Smets & Kennes, 1994) is utilized to fuse the INS and GPS data thereby delivering the accurate and high rate navigation parameter. The main advantage of using the DS lies in the fact that it does not assign weights to ignorant states but assigns the remaining weights to the unknown states (Bhattacharya, 2000; Bloc, 1996). Also, unlike the Bayesian theory, the probability of an event is not restricted to either an abnormal or the normal state. On the other hand, SVM can effectively model the highly non-linear input–output functional relationship due to its improved ability to avoid local minima (Bhatt, Aggarwal, Devabhaktuni, & Bhattacharya, 2012). The study thus utilizes SVM to

model the INS error during the GPS signal availability. In the case of outages, the trained SVM predicts the error in the INS solution and thus an accurate navigation solution is obtained while bridging the GPS outages.

The paper is organized into the following sections. Section 2 gives an overview of the DS theory and SVM. Section 3 explains the detailed implementation of the proposed DS-SVM algorithm. Section 4 presents the results of the DS-SVM model and its comparison with the existing ANN and Random Forest Regression (RFR) based PUA technique while Section 5 presents the concluding remarks.

## 2. Overview of Dempster Shafer theory and Support Vector Machines

Dempster Shafer theory was first introduced by Dempster in the 1960s, and was later extended by Shafer (1976). On one hand, DS theory represents a belief over a distinct piece of evidence with the help of a mass function (i.e., a Basic Belief Assignment (BBA)). On the other hand, DS theory attains the goal of data fusion by combining the belief using combination rule.

Let the frame of discernment be defined as  $\Omega = \{w_1, \dots, w_c\}$ , assumed to be a finite set of mutually exclusive and exhaustive events. The power set  $2^\Omega$  represents all the possible combinations of the element of  $\Omega$  (for example,  $w_1 \cup w_2$ ). The mass function or basic probability assignment ( $m$ ) maps the power set to the closed interval  $[0, 1]$ , such that Eqs. (1) and (2) are satisfied where  $\emptyset$  is an empty set and  $m(A)$  measures the degree of belief or evidence assigned to subset  $A$ .

$$\sum_{A \subseteq \Omega} m(A) = 1 \quad (1)$$

$$m(\emptyset) = 0 \quad (2)$$

For any  $A \subseteq \Omega$ ,  $m(A)$  represents the belief that could be exactly committed to  $A$ . The subset  $A$  of  $\Omega$  for which  $m(A) > 0$  are called a focal element. Belief and Plausibility associated with mass  $m$  is defined as

$$Bel(A) = \sum_{B \subseteq A} m(B) \quad (3)$$

$$Pl(A) = \sum_{B: A \cap B \neq \emptyset} m(B) \quad (4)$$

Belief,  $Bel(A)$  represents the degree to which we believe that the trueness is in  $A$  whereas the plausibility,  $Pl(A)$  indicates the amount of belief that could be potentially placed on  $A$ , if further information became available (Vapnik, 1999).  $Pl$  and  $Bel$  represents the upper and lower limit over the probability mass  $m$ .

To combine the two probability mass  $m_1$  and  $m_2$  on  $\Omega$ , obtained from two pieces of evidence Dempster's rule of combination is applied and is defined as:

$$m(A) = \frac{\sum_{B: C=A} m_1(B)m_2(C)}{\sum_{B: C \neq \emptyset} m_1(B)m_2(C)} \quad (5)$$

The new BBA represents the combined confidence measure that can be placed over  $A$ , derived from two distinct pieces of evidence. In Eq. (5), denominator corresponds to the value of conflict that avoids the assignment of nonzero probability mass to the null element.

### 2.1. Application to INS and GPS data fusion

Let us assume that the GPS and INS correspond to the two distinct pieces of evidence. Now, according to the DS theory, given a

mass function  $m_1(GPS)$  and  $m_2(INS)$ , the combined confidence measure over each of the two system i.e., INS and GPS defined as  $m_{GPS}$  and  $m_{INS}$  can be derived using DS combination rule based on both new and old available evidence and is given by (6) and (7).

$$m_{GPS} = 1/1 - K \sum_{GPS \cap INS = GPS} m_1(GPS)m_2(INS) \tag{6}$$

$$m_{INS} = 1/1 - K \sum_{GPS \cap INS = INS} m_1(GPS)m_2(INS) \tag{7}$$

Here,  $K$  represents the value of conflict and is given as  $K = \sum_{GPS \cap INS = \emptyset} m_1(GPS)m_2(INS)$ . The coefficient  $1/(1 - K)$  is a normalization factor whose role is to avoid assigning non-zero probabilities to the empty set in the combination (Shafer, 1976). Eq. (6) and (7) follows from (5), where the constant B and C corresponds to GPS and INS and A can be either GPS or INS depending upon the entity over which confidence measure needs to be derived. For an illustration, consider the DS combination rule applied to INS and GPS data as shown in Table 1.

After applying combination rule over GPS and INS data the combined confidence measure derived from Table 1 are given by (8) and (9).

$$m_{GPS} = \{m_1(GPS)m_2(GPS) + m_1(GPS \cup INS)m_2(GPS) + m_1(GPS)m_2(GPS \cup INS)\}/(1 - K) \tag{8}$$

$$m_{INS} = \{m_2(INS)m_1(INS) + m_1(GPS \cup INS)m_2(INS) + m_1(INS)m_2(GPS \cup INS)\}/(1 - K) \tag{9}$$

Now, once the confidence measure in each of the INS and GPS measurements is derived, the fused high data rate output is taken as the weighted sum of INS and GPS measurements. Here, the weight corresponds to the confidence measurements.

### 2.2. Support Vector Machine

Support Vector Machines as described in Vapnik (1999), Cortes and Vapnik (1995) have shown to deliver a promising solution in various classification and regression tasks due to its ability to avoid local minima, improved generalization capability, and sparse representation of the solution. SVM are based on Structural Risk Minimization (SRM) principle and thus tries to control the upper bound of generalization risk while reducing the model complexity. They do not suffer from over fitting problem and local minimization issues and hence offer improved generalization capability. In this study, a special form of SVM i.e., Support Vector Regression (SVR) is utilized for modeling the input–output functional relationship or regression purpose and is explained next.

Given a set of input–output sample pairs  $\{(\mathbf{x}_1, \mathbf{y}_1), (\mathbf{x}_2, \mathbf{y}_2), \dots, (\mathbf{x}_n, \mathbf{y}_n)\}$ , the objective of Nu-SVR technique is to approximate the non-linear relationship given in (10), such that  $f(x)$  should be as close as possible to the target value  $\mathbf{y}$  and should be as flat as possible in order to avoid over-fitting.

$$f(x) = \mathbf{w}^T \cdot \Phi(x) + b \tag{10}$$

In (10),  $\mathbf{w}^T$  is the weight vector,  $b$  is the bias and  $\Phi(x)$  represents the transformation function that maps the lower dimensional input

**Table 1**  
Dempster Shafer Combination rule.

$m_2$	$m_1$		
	GPS	INS	GPS $\cup$ INS
GPS	GPS	$\emptyset$	INS
INS	$\emptyset$	INS	INS
GPS $\cup$ INS	GPS	INS	GPS $\cup$ INS

space to a higher dimensional space. The primal objective of the problem thus reduces to (11), in order to ensure that the approximated function meets the above two objectives of closeness and flatness.

$$\text{minimize } \frac{1}{2} \|\mathbf{w}\|^2 + C \left\{ \gamma \cdot \varepsilon + \frac{1}{n} \sum_{i=1}^n (\xi + \xi^*) \right\};$$

subject to the constraints

$$\begin{aligned} y_i - \langle \mathbf{w}^T \cdot \Phi(x) \rangle - b &\leq \varepsilon + \xi_i^*, \\ \langle \mathbf{w}^T \cdot \Phi(x) \rangle + b - y_i &\leq \varepsilon + \xi_i^*, \\ \xi_i^*, \xi_i &\geq 0. \end{aligned} \tag{11}$$

where  $\varepsilon$  is a deviation of a function  $f(x)$  from its actual value and,  $\xi, \xi_i^*$  are additional slack variables introduced by Cortes & Vapnik, 1995, which determines that, deviations of magnitude  $\xi$  above  $\varepsilon$  error are tolerated. The constant C known as regularization parameter determines the tradeoff between the flatness of  $f$  and tolerance of error above  $\varepsilon$ . Further  $\Upsilon$  ( $0 \leq \Upsilon \leq 1$ ), represents the upper bound on the function of margin errors in the training set and establishes the lower bound on the fraction of support vectors.

To solve the primal problem in (11), its dual formulation is introduced by constructing Lagrange function (L) given as:

$$\begin{aligned} L : \frac{1}{2} \|\mathbf{w}\|^2 + C \left\{ \Upsilon \cdot \varepsilon + \frac{1}{n} \sum_{i=1}^n (\xi + \xi^*) \right\} &- \frac{1}{n} \sum_{i=1}^n (\eta \cdot \xi + \eta^* \cdot \xi^*) \\ &- \frac{1}{n} \sum_{i=1}^n (\varepsilon + \xi_1 + y_i - \mathbf{w}^T \cdot \Phi(x) - b) - \frac{1}{n} \sum_{i=1}^n (\varepsilon + \xi_i - y_i \\ &+ \mathbf{w}^T \cdot \Phi(x) + b) - \beta \cdot \varepsilon. \end{aligned} \tag{12}$$

where  $\alpha, \alpha^*, \eta, \eta^*, \beta$  are Lagrange multipliers and  $\alpha^{(*)} = \alpha \cdot \alpha^*$ . Thus, maximizing the Lagrange function gives  $w = \sum_{i=1}^n (\alpha_i - \alpha_i^*) \cdot \Phi(x_i)$  and yields the dual optimization problem:

$$\begin{aligned} \text{maximizes } &- \frac{1}{2} \sum_{i,j=1}^n (\alpha_i - \alpha_i^*) \cdot (\alpha_j - \alpha_j^*) \cdot K(x_i, x_j) + \sum_{i=1}^n y_i \cdot (\alpha_i - \alpha_i^*); \\ \text{subject to } &\sum_{i=1}^n (\alpha_i - \alpha_i^*) = 0, \\ &\sum_{i=1}^n (\alpha_i + \alpha_i^*) \leq C \Upsilon, \\ &\alpha_i, \alpha_i^* \in \left[ 0, \frac{C}{n} \right]. \end{aligned} \tag{13}$$

where  $K(x_i, x_j)$  denotes the kernel function given by  $K(x_i, x_j) = \Phi(x_i)^T \cdot \Phi(x_j)$ . The solution to (13) yields the Lagrange multipliers  $\alpha, \alpha^*$ . Substituting weight  $\mathbf{w}$  in (10), the approximated function is given as:

$$f(x) = \sum_{i=1}^n (\alpha_i - \alpha_i^*) \cdot K(x_i, x) + b. \tag{14}$$

Depending on the problem complexity, the choice of kernel varies. Usually four different types of kernel are in use, i.e., polynomial function, Radial Basis Function (RBF), sigmoid function and linear function. However, the selection of an appropriate kernel determines the model prediction accuracy. In our study, we selected RBF kernel as it delivers an acceptable accuracy and has less implementation difficulties (Keerthi & Lin, 2003). The parameter  $b$  is identified using Karush–Kuhn–Tucker conditions (Karush, 1939; Kuhn & Tucker, 1951). For further details related to Nu-SVR working, please refer to Alex and Scholkopf (2004) and Hu, Che, and Cheng (2009).

Thus, Nu-SVR approach identifies the Lagrange multipliers  $\alpha$ ,  $\alpha^*$  and  $b$  for a given input–output training sample pairs. After parameter identification, the model can be utilized to predict the output corresponding to an unknown input using (14). In our study, Nu-SVR is utilized to model the time varying INS sensor errors and is explained in next section.

### 3. Proposed DS-SVM methodology

When GPS signals are available, DS fusion theory effectively fuses the data coming from the INS and GPS units. The DS theory estimates the confidence measure derived from individual unit (i.e., INS and GPS) mass functions (Eqs. (15), (16)) in order to effectively decide whose measurement i.e., INS or the GPS should be given more weightage thereby delivering an accurate navigation solution.

$$m_1(GPS) = \frac{1}{(2\pi)^{1/2} C_{GPS}} \exp \left[ -0.5(GPS - \mu_{GPS})^T C_{GPS}^{-1} (GPS - \mu_{GPS}) \right] \tag{15}$$

$$m_2(INS) = \frac{1}{(2\pi)^{1/2} C_{INS}} \exp \left[ -0.5(INS - \mu_{INS})^T C_{INS}^{-1} (INS - \mu_{INS}) \right] \tag{16}$$

where  $C_{GPS}$ ,  $C_{INS}$  are the covariances of GPS and INS data while  $\mu_{GPS}$ ,  $\mu_{INS}$ , are the mean values of GPS and INS measurements respectively. Here, we assumed the distributions to be Gaussian as per the central limit theorem as part of this initial innovative research.

Thus DS theory has effectively combined INS and GPS output as long as GPS is available in this study. However in the absence of GPS signals, the DS theory assigns the 100% confidence to the INS, whose output is corrupted with noises and thus solution starts drifting with time. To overcome this drift in the standalone INS solution, as explained, Nu-SVR is utilized that models the INS errors. Thus, in the case of outages, the trained SVR model predicts and compensates the INS error and hence improves positioning accuracy.

For the INS and GPS data integration, a loosely coupled integration strategy is adopted where the processed GPS measurements is fused with INS for its error computations Godha (2006). However, two implementation approaches exist for the strategy i.e., open loop and closed loop. The former approach i.e., open loop estimates the time varying INS error using GPS information without updating the INS solution. On the other hand, the latter approach i.e., closed loop estimates and updates the INS frequently using GPS information. In this study, loosely coupled integration strategy is implemented using closed loop approach as it continuously updates the INS every epoch, thereby suppressing the time growing low-cost INS error (Godha, 2006). Fig. 1 illustrates the adopted integration strategy in a closed loop approach.

As shown in Fig. 1, the INS and GPS data is fused using DS theory in closed loop configuration. The fused output is fed back to the

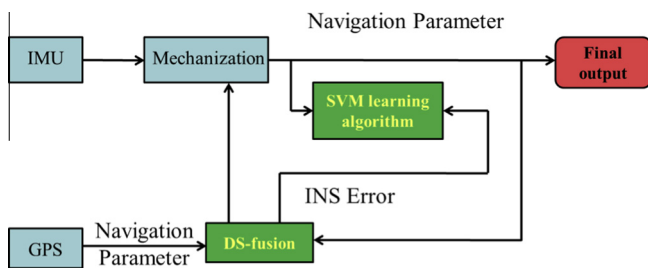


Fig. 1. Closed loop system configuration under no GPS outages.

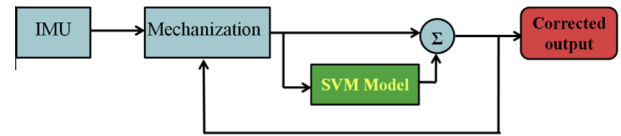


Fig. 2. Closed loop system configuration during GPS outages.

mechanization process where it acts as a reference solution to derive the navigation parameter for the next epoch. Simultaneously, an estimate of the INS error is taken as the SVM desired output and the INS output as the input (demonstrated in Fig. 1). In this study, both, the errors in the position and velocity components along the three directions can be modeled. However, to reduce the integration complexity only errors in the INS velocity components along three directions are modeled as it avoids introducing additional SVM network. This whole process of fusion using DS theory and error modeling through SVM is continued during GPS signal availability. During outages, the trained SVM predicts and compensates the INS error thereby improving the standalone INS accuracy, as shown in Fig. 2.

A step by step algorithm for the INS and GPS data fusion using DS theory is explained next:

#### Algorithm DS-SVM methodology working procedure

##### Repeat

- Step 1:** Obtain the probability mass corresponding to each of the sensor measurements, i.e., INS and GPS;
- Step 2** Evaluate the fused output, i.e., weighted sum of the INS and GPS measurements: acc to eqs.15 and 16.
- Step 3:** Obtain the error, defined as the difference between INS and GPS solution;
- Step 4:** Train the Nu-SVR model using INS solution as input and the obtained error as output;

Until GPS outage occur; else

- Step 5:** Estimate the error using INS solution as input to the trained Nu-SVR model;
- Step 6:** Compensate the INS solution using the estimated error derived in step 4 to obtain the accurate navigation.

The proposed algorithm effectively bridges the period of GPS outages because of the enhanced generalization ability of SVM. The validity of the proposed method is tested by using real field test data collected using a low-cost IMU and a DGPS unit; under both GPS outages and no GPS outages conditions.

### 4. Results

The amount of reduction in the positional error drift against existing technique demonstrates the effectiveness of the proposed

Table 2  
Characteristics of Crossbow IMU and HG 1700.

	Crossbow IMU 300CC	HG 1700
<i>Gyroscope</i>		
Bias	<± 2.0 °/s	1.0 °/h
Scale factor	<1%	150 ppm
Random walk	<2.25 °/√h	0.12 °/√h
<i>Accelerometer</i>		
Bias	±30.0 mg	1.0 mg
Scale factor	<1%	300 ppm
Random walk	<0.15 m/s/√h	0.019 m/s/√h

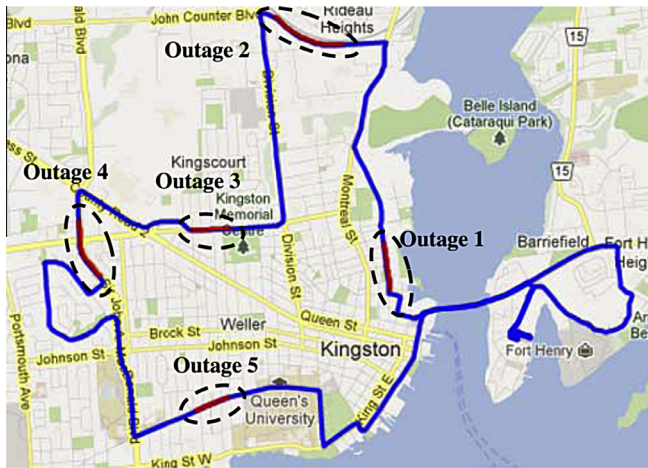


Fig. 3. Field test trajectory depicting simulated GPS outages (in Red). For interpretation of color in figure, the reader is referred to the web version of this book.

algorithm in bridging the GPS outages. In our study, we have compared the performance of DS-SVM approach with the Position Update Architecture (utilizing three layer multi-layer perceptrons) neural network, and a RFR method to integrate the INS and GPS data as explained in Adusumilli, Bhatt, Wong, Bhattacharya, and Devabhaktuni (2013), El-Sheimy et al. (2006). The field test data was collected using Crossbow IMU 300CC-100, reference high grade IMU by Honeywell (HG 1700), Novatel OEM GPS receivers and a computer. The IMU data collection rate was 100 Hz and the specifications is shown in Table 2.

Fig. 3 illustrates the field test trajectory that comprises of all the real-life scenarios encountered by a typical land vehicle which includes high speed highway section, suburban roads with hills, trees and winding turns, urban streets with frequent stops and sudden vehicle accelerations/decelerations.

To evaluate the proposed DS-SVM method against existing ANN and RFR based PUA five simulated GPS outages of different durations i.e., 30 and 40 s each are considered under diverse conditions such as straight portions, turns, slopes, high speed, and slow speeds (shown in Fig. 3).

To achieve the better accuracy using proposed DS-SVM, requires an optimal selection of cost parameter ( $C$ ), gamma ( $\gamma$ ) and Nu ( $\nu$ ) associated with Nu-SVR model. However, to reduce the implementation complexity only the hyper-parameter gamma ( $\gamma$ ) is varied, keeping other constant throughout the training process. Nu-SVR utilized, is highly accurate in modeling the input–output functional relationship and achieved a training goal of mean square error (MSE) to be less than  $10^{-3}$ . In the absence of outages, the DS based fusion provides an accurate high data rate output whereas in case of outages, the trained Nu-SVR model predicts and compensates the INS sensor error. Table 3 below depicts the optimal parameter obtained before each of the five simulated outages.

The PUA model considered in this study utilizes INS velocity and azimuth as input and the position coordinate differences between two consecutive epochs (taken from GPS) as the desired output. Thus, the PUA model based on ANN and RFR is trained as long as the GPS signals are available whereas in the case of outages, the trained model is utilized to predict the position coordinates difference. For further details please refer to Adusumilli et al. (2013), Bhatt, Aggarwal, Devabhaktuni, et al. (2012), El-Sheimy et al. (2006). The ANN is trained using quasi-Newton training algorithm because of its faster convergence ability (Dennis & Schnabel, 1983; Likas & Stafylopatis, 2000).

The model performance parameter is evaluated with Root Mean Square Error (RMSE), given in (17), by comparing the predicted

Table 3  
Values of gamma associated with Nu-SVR model.

	$\gamma_{\text{Vel}_N}$	$\gamma_{\text{Vel}_E}$	$\gamma_{\text{Vel}_D}$
1st Outage	1e-07	9e-11	3e-03
2nd Outage	1e-12	5e-04	1e-05
3rd Outage	1e-07	1e-04	1e-08
4th Outage	1e-05	1e-10	1e-02
5th Outage	1e-10	25e-08	1e-04

position components obtained using the proposed methodology and the existing model with the reference solution.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N [\hat{y}_p(\mathbf{x}_i, \mathbf{w}) - y_p]^2}{N}} \quad (17)$$

where  $\hat{y}_p$  and  $y_p$  are the predicted and the desired output and  $N$  corresponds to the GPS outage duration.

As explained, the proposed DS-SVM methodology accurately fuses the INS and GPS data and models the INS error during GPS signal availability. Thus, the fusion and training process continues prior to the outages occurrence considered in this research work. Fig. 4 illustrates the drift in the positional error during 1st GPS outage of duration 40 s. DS-SVM methodology and RFR predicted position component (in brown and green) possess less drift in comparison to ANN (in violet). Moreover, the percentage improvement in the positioning accuracy was found to improve by 20% and 18% against ANN and RFR. During GPS availability, all the three method demonstrates similar performance as evident from the figure below.

Similarly, for the second outage, when the vehicle moves along the curve the predicted trajectory was quite close to the reference trajectory (in blue) as shown in Fig. 5 and thus effectively reduces the drift in the positional error with an 87% and 81% improvement against ANN and RFR. This improvement is mainly attributed to the

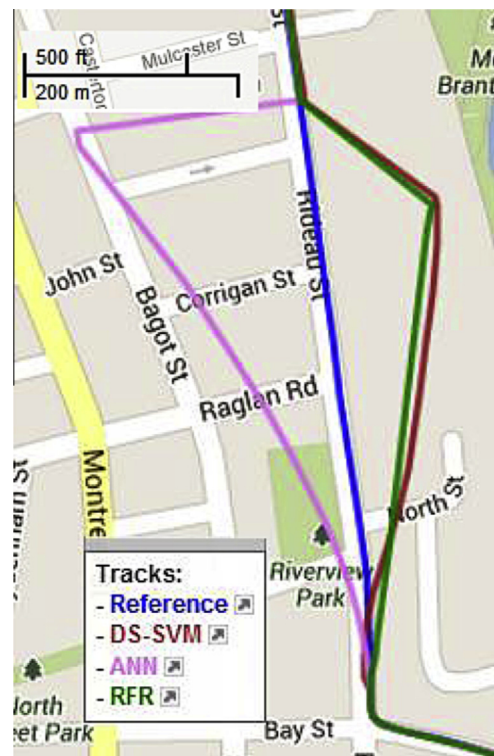


Fig. 4. Performance during GPS outage 1.



Fig. 5. Performance during GPS outage 2.

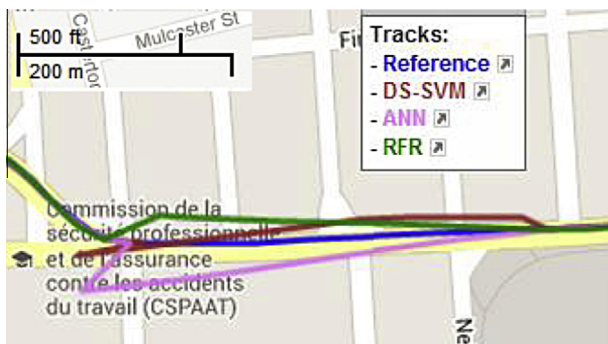


Fig. 6. Performance during GPS outage 3.

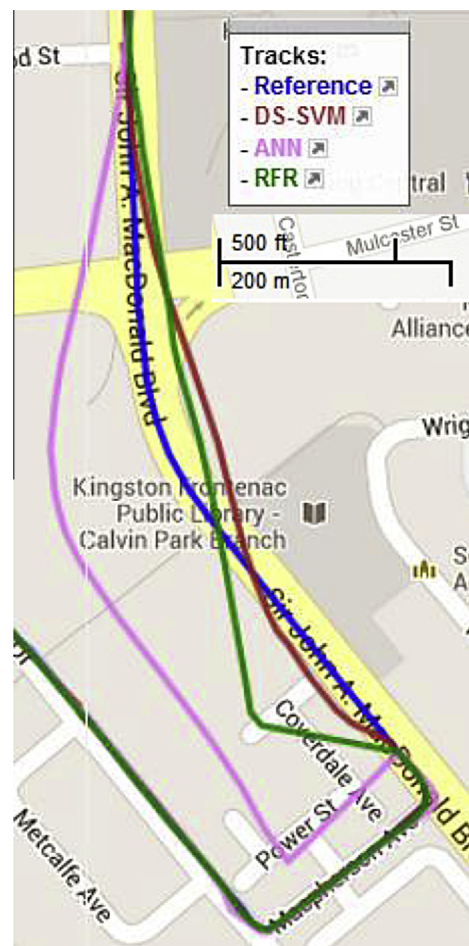


Fig. 7. Performance during GPS outage 4.

trained Nu-SVR ability to effectively model the INS velocity errors. These corrected velocity components are then fed to the mechanization process to deliver the position and velocity components at the next epoch.

The third outage corresponds to the portion of the trajectory for the vehicle motion along a straight line. During this period the vehicle has a zero acceleration along north direction whereas non-zero acceleration along east direction. Fig. 6 depicts the predicted and the reference trajectory. For a 30 s GPS outage, DS-SVM produces an rmse of 22.03 m whereas the ANN and RFR based PUA resulted in an rmse of 44.55 m and 30.23 m. The percentage improvement in the positional error reduction is found to improve by 51% and 27%.

The fourth outage that last for 40 s is taken along a curve where the vehicle moves with zero acceleration along the north direction and a constant acceleration along east. The total percentage improvement in the positional accuracy is found to improve by 65% and 23% against ANN and RFR as illustrated in Fig. 7.

Further, the proposed methodology showed a similar improvement for the fifth GPS outage that lasts for 30 s. The reference trajectory (in blue) and the predicted trajectory obtained using DS-SVM (in brown), ANN and RFR (in violet and green) is as depicted in Fig. 8.

For all the five simulated GPS outages considered in this study, the DS-SVM algorithm was effectively able to reduce the time growing positional error associated with standalone INS solution. A quantitative comparison of the accumulated position error using our proposed DS-SVM algorithm against conventional ANN and RFR based PUA model is shown in Table 4. In Table 4, the highlighted column corresponds to the least positional error obtained using DS-SVM methodology.

As is evident from Figs. 4–8, the navigation accuracy of the proposed DS-SVM model was found to improve against conventional ANN and RFR based PUA model by 20–87%. The grade of INS considered in this study is a low-grade INS, resulting in a huge positional drift within a short time interval if left uncompensated. Though the proposed algorithm delivers better accuracy, but the system accuracy degrades for a long duration of GPS outages (i.e., 50 s). This is due to the algorithm implementation in a closed loop mode. Therefore, the study did not consider long period of GPS outages that may have resulted in huge positional drift due to

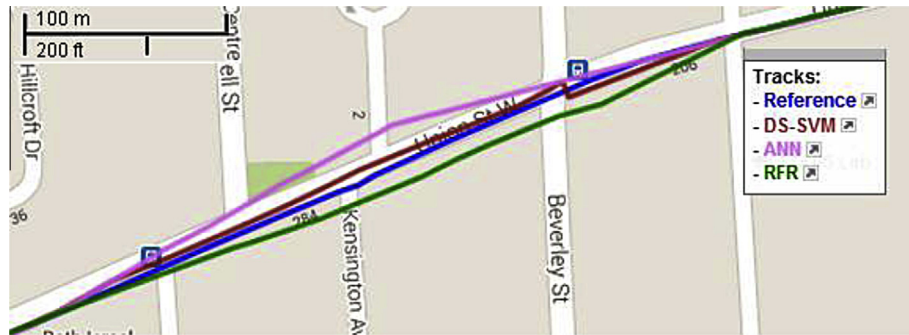


Fig. 8. Performance during GPS outage 5.

**Table 4**  
Position errors for the proposed DS-SVM model and the conventional PUA model.

	GPS outage length (m)	Total positional error (m)		
		PUA	RFR	DS-SVM
Outage 1 (40 s)	472	80.82	77.33	<b>62.87</b>
Outage 2 (40 s)	732	144.8	103.49	<b>19.07</b>
Outage 3 (30 s)	362	44.55	30.23	<b>22.03</b>
Outage 4 (40 s)	597	91.10	40.99	<b>31.47</b>
Outage 5 (30 s)	336	85.79	38.49	<b>27.78</b>

low-grade of INS utilized. However, the DS-SVM model could deliver better navigation accuracy for a medium grade INS during long GPS outages.

Beside the significant reduction in the positional error against PUA model, the proposed DS-SVM methodology also offers the velocity information along with the position components during GPS outages.

## 5. Conclusions

This paper introduces a novel and hybrid fusion methodology known as DS-SVM with the aim of improving the standalone low-cost INS accuracy to bridge the period of GPS outages. During the GPS signal availability, the DS based fusion theory accurately fuses the INS and GPS data and simultaneously models INS error using SVM. In the case of outages, the trained SVM model is utilized to predict and compensate the INS error thereby delivering an accurate and reliable navigation solution. The proposed algorithm showed 20–87% improvement in the positioning accuracy corresponding to different GPS outages considered in this study. In conclusion, the study fulfills the basic aim of improving the standalone INS accuracy and provides continuous navigation solution with and without GPS signals.

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