History of Fractional Calculus

2010

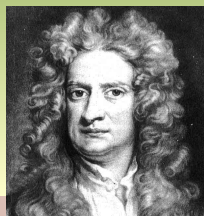### J. Tenreiro Machado, Virginia Kiryakova, Francesco Mainardi

Leibniz first introduced the idea of a symbolic method and used the symbol $d^n y / dx^n = D^n y$ for the nth derivative, where n is a non-negative integer.

Gottfried Leibniz
1646 -1716

In a letter to L`Hôpital in 1695 Leibniz raised the following question: "Can the meaning of derivatives with integer order be generalized to derivatives with non-integer orders?" L`Hôpital was somewhat curious about that question and replied by another question to Leibniz: *"What if the order will be 1/2?"* Leibniz in a letter dated September 30, 1695 - the exact birthday of the fractional calculus! — replied: *"It will lead to a paradox, from which one day useful consequences will be drawn."*

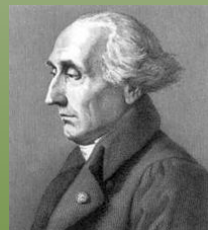Guillaume de l'Hôpital
1661- 1704

John Wallis
1616-1703

Historically, Isaac Newton and Gottfried Leibniz independently discovered calculus in the 17th century.

Isaac Newton
1643-1727
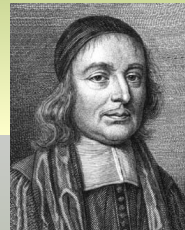
Leonhard Euler
1707-1783

Euler observed that the result of the evaluation of $d^n y / dx^n$ of the power function x^p has a meaning for non-integer p.

A relevant part of the history of fractional calculus began with the papers of Abel and Liouville.
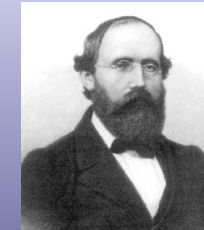
Niels Abel
1802-1829

Joseph Liouville
1809-1882

Liouville formally extended the formula for the derivative of integral order $D^n e^{ax}$ to derivatives of arbitrary order α $D^\alpha e^{ax} = a^\alpha e^{ax}$
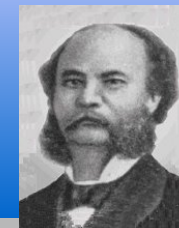
Bernhard Riemann
1826-1866

In a paper written when just a student Riemann, that was published only ten years after is death, he arrived to an expression for fractional integration that became one of the main formulae together with Liouville construction.

Laplace proposed the idea of differentiation of non-integer order for functions representable by an integral $\int T(t) t^{-x} dt$

Pierre-Simon Laplace
1749-1827

Joseph-Louis Lagrange
1736-1813

Fourier suggested the idea of using his integral representation of $f(x)$ to define the derivative for non-integer order.

Joseph Fourier
1768-1830

Anton Grünwald
1838-1920

Aleksey Letnikov
1837-1888

Grünwald and Letnikov developed an approach to fractional differentiation based on the limit of a sum
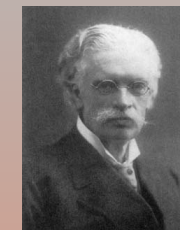
Karl Weierstrass
1815-1897

Hermann Weyl
1885-1955

Harold Davis
1892-1974

Magnus Mittag-Leffler
1846-1927

Godfrey Hardy
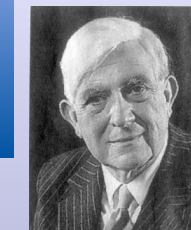1877-1947

John Littlewood
1885-1977

Marcel Riesz
1886-1969

William Feller
1906-1970

Ian Sneddon
1919-2000

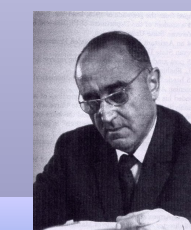Hjalmar Holmgren
1822-1885

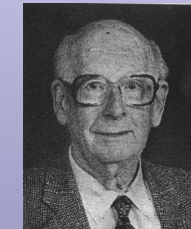Salvatore Pincherle
1853-1936

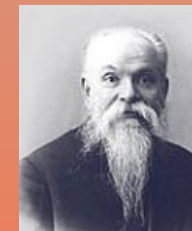Andrew Gemant
1895 - 1983

Yury Rabotnov
1914-1985

Eric Love
1912-2001

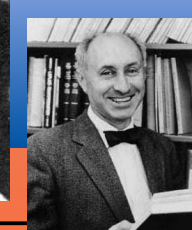Pavel Nekrasov
1853-1924

Paul Lévy
1886-1971

Mkhtar Djrbashjan
1918-1994
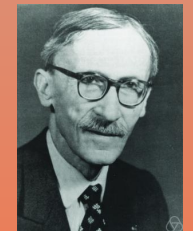
Hermann Kober
1888-1973

Arthur Erdélyi
1908-1977

Antoni Zygmund
1900-1992

Anatoly Kilbas
1948-2010

Oliver Heaviside
1850-1925

Jacques Hadamard
1865-1963

1700   1750   1800   1850   1900   1950