Journal of Vibration and Control

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Journal of Vibration and Control 2008; 14; 1389
DOI: 10.1177/1077546307087442

Fractional Electrical Impedances in Botanical Elements

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(Received 21 December 2005; accepted 4 October 2006)

Abstract: Fractional calculus (FC) is no longer considered solely from a mathematical viewpoint, and is now applied in many emerging scientific areas, such as electricity, magnetism, mechanics, fluid dynamics, and medicine. In the field of dynamical systems, significant work has been carried out proving the importance of fractional order mathematical models. This article studies the electrical impedance of vegetables and fruits from a FC perspective. From this line of thought, several experiments are developed for measuring the impedance of botanical elements. The results are analyzed using Bode and polar diagrams, which lead to electrical circuit models revealing fractional-order behaviour.

Keywords: Fractional calculus, electrical impedance, fruits, vegetables, modelling

1. INTRODUCTION

Fractional calculus (FC) is a generalization of integration and differentiation to non-integer orders. The fundamental operator is ${}_aD_t^{\alpha}$, where the order α is a real or complex number, and the subscripts a and t are the two limits related to the operation (Oldham and Spanier, 1974; Samko et al., 1993; Oustaloup, 1995; Miller, 2002).

Recent studies have brought FC to more widespread attention, revealing that many physical phenomena can be modeled through fractional differential equations, and fractional-order systems have been a subject of increasing interest (Tenreiro Machado and Jesus, 2004; Jesus et al., 2006a,b). The importance of fractional order mathematical models is that they can be used to produce a more accurate description, and so give a deeper insight into the physical processes underlying long range memory behaviours.

Capacitors are crucial elements in many integrated circuits and, are used extensively in many electronic systems (Samavati et al., 1998). Jonscher (1993) demonstrated that the ideal capacitor cannot exist in nature, because an impedance of the form $1/(j\omega C)$ would violate causality (Bohannan, 2002). The dielectric material exhibits a realistic fractional behaviour for impedances $1/(j\omega C_F)^{\alpha}$, with $\alpha \in \Re^{+}$.

Journal of Vibration and Control, **14(9–10):** 1389–1402, 2008 DOI: 10.1177/1077546307087442

Bearing these ideas in mind, this article analyzes the fractional-order modelling of botanical electrical impedances and is organized as follows: Section 2 presents the fundamental electrical concepts. Sections 3 and 4 describe the research experiments, for the identification of the impedances, and the fractional model of the electrical impedance, respectively. Finally, Section 5 describes the main conclusions.

2. ELECTRICAL IMPEDANCE

In an electrical circuit, the voltage $u(t)$ and current $i(t)$ can be expressed as a function of the time t

$$u(t) = U_0 \cos(\omega t) \tag{1}$$

$$i(t) = I_0 \cos(\omega t + \phi) \tag{2}$$

where U_0 and I_0 are the amplitudes of the signals, ω is the angular frequency, and ϕ is the current phase shift. The voltage and current can be expressed in complex form as

$$u(t) = \text{Re}\left\{U_0 e^{j(\omega t)}\right\} \tag{3}$$

$$i(t) = \text{Re}\left\{I_0 e^{j(\omega t+\phi)}\right\}. \tag{4}$$

Consequently, in complex form the electrical impedance $Z(j\omega)$ is given by the expression

$$Z(j\omega) = \frac{U(j\omega)}{I(j\omega)} = Z_0 e^{j\phi}. \tag{5}$$

In fact, however, in modelling an electrochemical phenomenon, a constant phase element (CPE) is often used, as the surface is not homogeneous (Barsoukov and Macdonald, 2005). With a CPE we have the complex expression

$$Z(j\omega) = \frac{1}{(j\omega C_F)^{\alpha}} \tag{6}$$

where C_F is a fractional order capacitance and the fractional order α is a parameter that can change between 0 and 1, giving an ideal capacitor when $\alpha = 1$. It should be noted that, for the CPE, the SI base units of the C_F element are $[m^{-2/\alpha} kg^{-1/\alpha} s^{(\alpha+3)/\alpha} A^{2/\alpha}]$, $0 < \alpha \leq 1$.

It is well known that, in electrochemical systems with diffusion, the impedance is modelled using the so-called Warburg element (Ho et al., 1980, Barsoukov and Macdonald, 2005). The Warburg element arises from one-dimensional diffusion of an ionic species to the electrode. If the impedance is under an infinite diffusion layer, the Warburg impedance is given by

$$Z(j\omega) = \frac{R}{(j\omega C_F)^{0.5}} \tag{7}$$

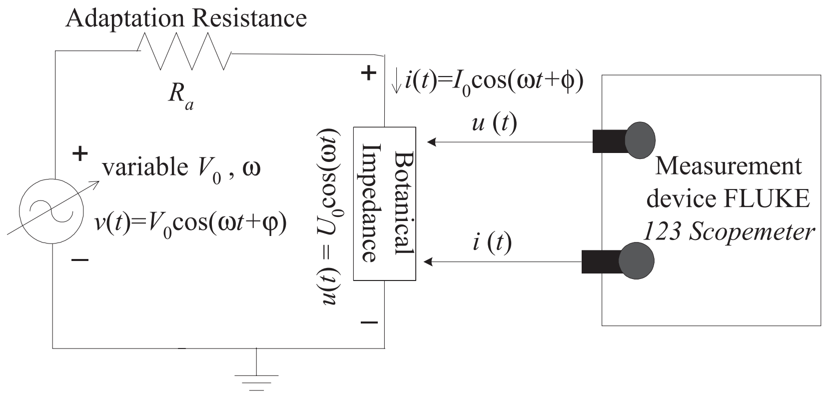

Figure 1. Electrical circuit for the measurement of the botanical impedance $Z(j\omega)$.

where R is the diffusion resistance. If the diffusion process has finite length, the Warburg element becomes

$$Z(j\omega) = R\frac{\tanh{(j\omega\tau)}^{0.5}}{(\tau)^{0.5}} \tag{8}$$

with $\tau = \delta^2/D$, where R is the diffusion resistance, τ is the diffusion time constant, δ is the diffusion layer thickness and D is the diffusion coefficient (Machowski et al., 2003).

3. STUDY OF FRACTIONAL ORDER ELECTRICAL IMPEDANCES

Fruits and vegetables are composed of cells that are sensitive to heat, pressure, and other stimuli. These systems constitute electrical circuits exhibiting complex behaviour. Bearing these facts in mind, we studied the electrical impedance for several botanical elements, from a FC point of view.

We applied sinusoidal excitation signals $v(t)$ to the botanical system (see Figure 1), for several distinct frequencies ω, and the impedance $Z(j\omega)$ was measured, based on the resulting voltage $u(t)$ and current $i(t)$. We also measured the environmental temperature, and the weight, length, and width of all the botanical elements, to help us understand how these factors influence $Z(j\omega)$. In addition, we developed several different experiments for evaluating the variation of the impedance $Z(j\omega)$ with changing amplitude of the input signal V_0, for different electrode penetrations (i.e., lengths inside the element) Δ, and for different environmental temperature T, weight W, and dimension (length times width) D.

The value of the adaptation resistance R_a was changed during the experiments, in order to adapt the values of the voltage and current to the scale of the measurement device.

We started by analyzing the impedance for an input signal with $V_0 = 10$ V, using a constant adaptation resistance $R_a = 15$ kΩ, applied to one *Solanum Tuberosum* (the common potato), with weight $W = 1.24 \times 10^{-1}$ kg, environmental temperature $T = 26.5$°C, dimension $D = 7.97 \times 10^{-2} \times 5.99 \times 10^{-2}$ m, and electrode length penetration $\Delta = 2.1 \times 10^{-2}$ m.

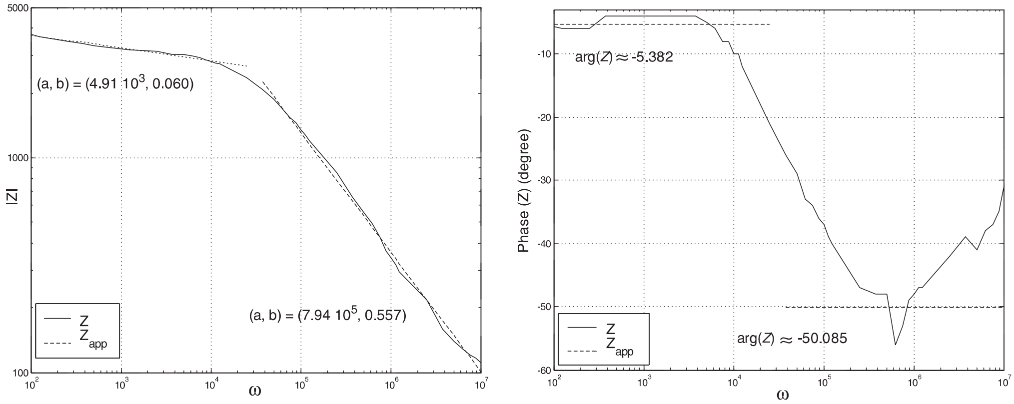

Figure 2. Bode diagrams of the impedance $Z(j\omega)$ for the *Solanum Tuberosum*.

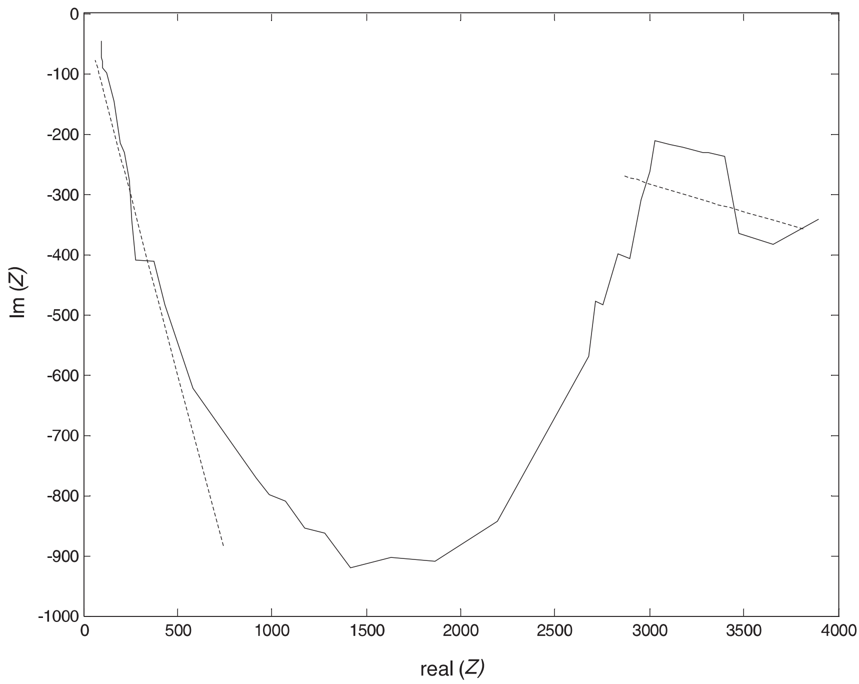

Figure 3. Polar diagrams of the impedance $Z(j\omega)$ for the *Solanum Tuberosum*.

Figure 2 shows the Bode diagrams for the resulting $Z(j\omega)$, and Figure 3 the corresponding polar plot. The results reveal that the system has a fractional order impedance. In fact, approximating the experimental results in the amplitude Bode diagram through a power function $|Z(j\omega)| = a\omega^{-b}$, we obtain $(a, b) = (4.91 \times 10^3, 0.0598)$ and $(a, b) = (7.94 \times 10^5, 0.5565)$ for the low and high frequency ranges, respectively.

Table 1. Comparison of the values of $|Z(j\omega)| \approx a\omega^{-b}$ for different amplitudes of the input signal.

Amplitude	Low ω		High ω	
[V]	a	b	a	b
5	4.79×10^3	0.062	6.52×10^5	0.542
10	4.91×10^3	0.060	7.94×10^5	0.557
15	4.54×10^3	0.054	5.66×10^5	0.530
20	4.65×10^3	0.055	5.86×10^5	0.530

In order to analyze the system's linearity, we evaluated $Z(j\omega)$ for different input system amplitudes, namely $V_0 = 5$ V, 10 V, and 20 V, maintaining the constant adaptation resistance $R_a = 15$ kΩ. The impedance $Z(j\omega)$ has a fractional order, and this characteristic does not change significantly with the variation of input signal amplitude (Table 1). Therefore, we can conclude that this system has a linear characteristic.

In a second experiment, we varied the length of electrode penetration Δ inside the *Solanum Tuberosum*, and evaluated its influence on the value of the impedance. The electrode was adjusted to $\Delta = 1.42 \times 10^{-2}$ m, with $V_0 = 10$ V and an adaptation resistance $R_a = 15$ kΩ, giving the $|Z(j\omega)|$ approximations $(a, b) = (5.48 \times 10^3, 0.0450)$ for low frequencies, and $(a, b) = (1.00 \times 10^6, 0.5651)$ for high frequencies. From this, we concluded that the length of wire inside the potato does not significantly change the values of fractional order b. In additon, the linearity was re-confirmed, with several experiments varying V_0 (results not shown).

The last experiment with the *Solanum Tuberosum* considered the variation of environmental temperature. Here, we use the *Solanum Tuberosum* and conditions from the first experiment, but with $T = 25.7$°C. The resulting impedance $|Z(j\omega)|$ values were $(a, b) = (8.91 \times 10^3, 0.0555)$ and $(a, b) = (7.10 \times 10^5, 0.5010)$, for low and high frequencies, respectively. Again, it can be seen that the variation of the fractional order b is small.

Another issue that may influence the results is the weight W. Therefore, we applied an input signal with amplitude $V_0 = 10$ V, with adaptation resistance $R_a = 15$ kΩ, environmental temperature $T = 26.5$°C, and electrode penetration $\Delta = 2.1 \times 10^{-2}$ m to another *Solanum Tuberosum* with different dimensions ($D = 7.16 \times 10^{-2} \times 3.99 \times 10^{-2}$ m) and weight ($W = 5.89 \times 10^{-2}$ kg). The asymptotic results for $|Z(j\omega)|$ are $(a, b) = (7.17 \times 10^3, 0.0546)$ and $(a, b) = (2.00 \times 10^6, 0.5990)$ for low and high frequencies, respectively. Again, this experiment does not reveal significant variation in the fractional order b; the linearity was also confirmed with other values of V_0.

In conclusion, the fractional order of the electrical impedance does not change significantly with the factors analyzed. In this line of thought, we organized similar experiments with other vegetables and fruits.

Table 2 presents the characteristics of the vegetables and fruits tested. The results shown are for an amplitude of input signal $V_0 = 10$ V and electrode penetration $\Delta = 2.1 \times 10^{-2}$ m. Figures 4 and 5 show $\mathrm{Re}\{Z(j\omega)\}$ and $-\mathrm{Im}\{Z(j\omega)\}$ for the vegetables and fruits (respectively) under study, and the corresponding approximation values. In these experiments, the adaptation resistance R_a is changed for each case (i.e., each species).

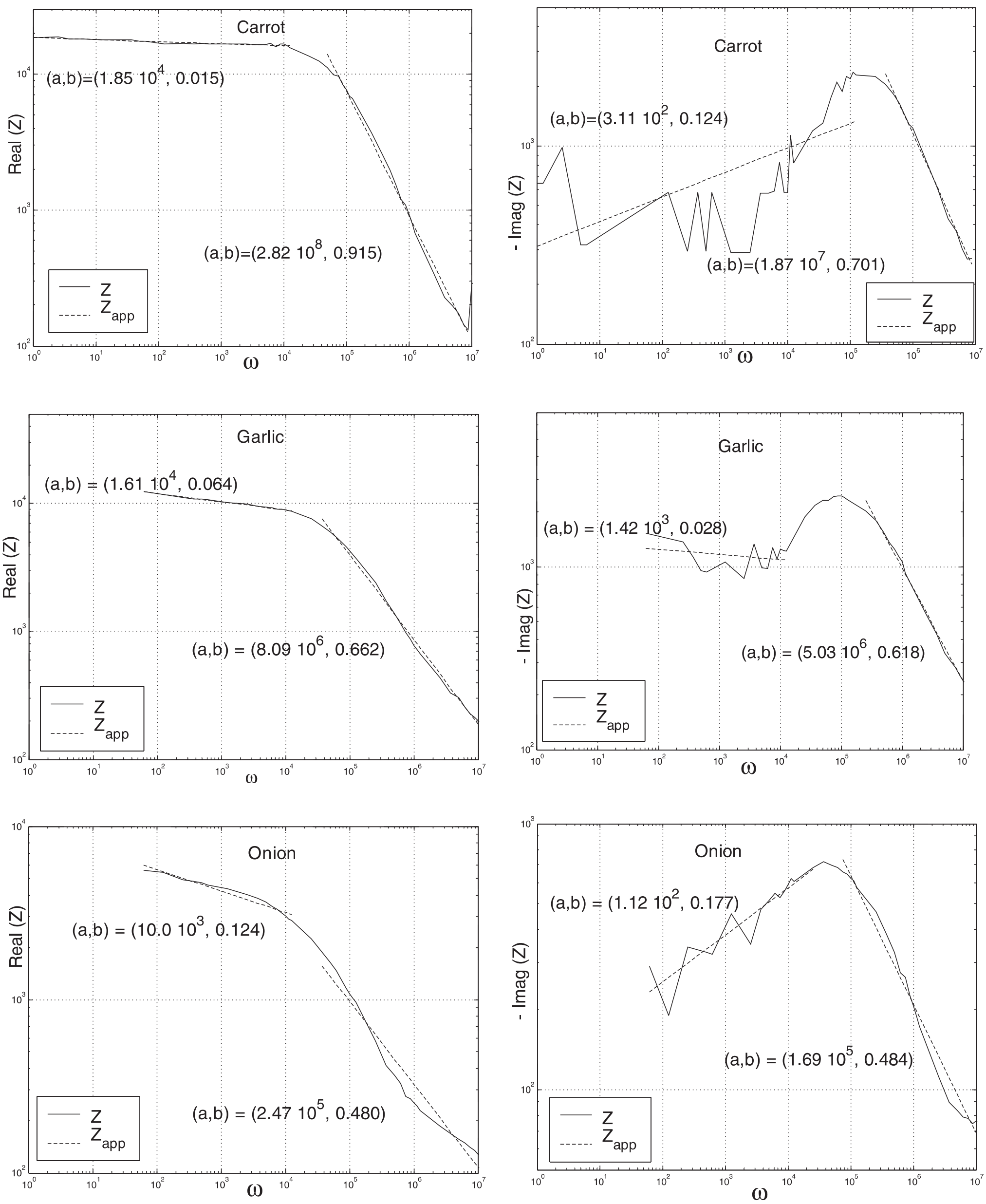

Figure 4. Diagrams of $\mathrm{Re}\{Z(j\omega)\}$ and $-\mathrm{Im}\{Z(j\omega)\}$ of the electrical impedance for several vegetables: Carrot (*Daucus Carota L.*) (with $R_a = 4.7\ \mathrm{k\Omega}$), garlic (*Allium sativum L.*) (with $R_a = 15.0\ \mathrm{k\Omega}$), onion (*Allium cepa L.*) (with $R_a = 2.7\ \mathrm{k\Omega}$), potato (*Solanum tuberosum*) (with $R_a = 15.0\ \mathrm{k\Omega}$), and turnip (*Brassica napobrassica*) (with $R_a = 2.2\ \mathrm{k\Omega}$).

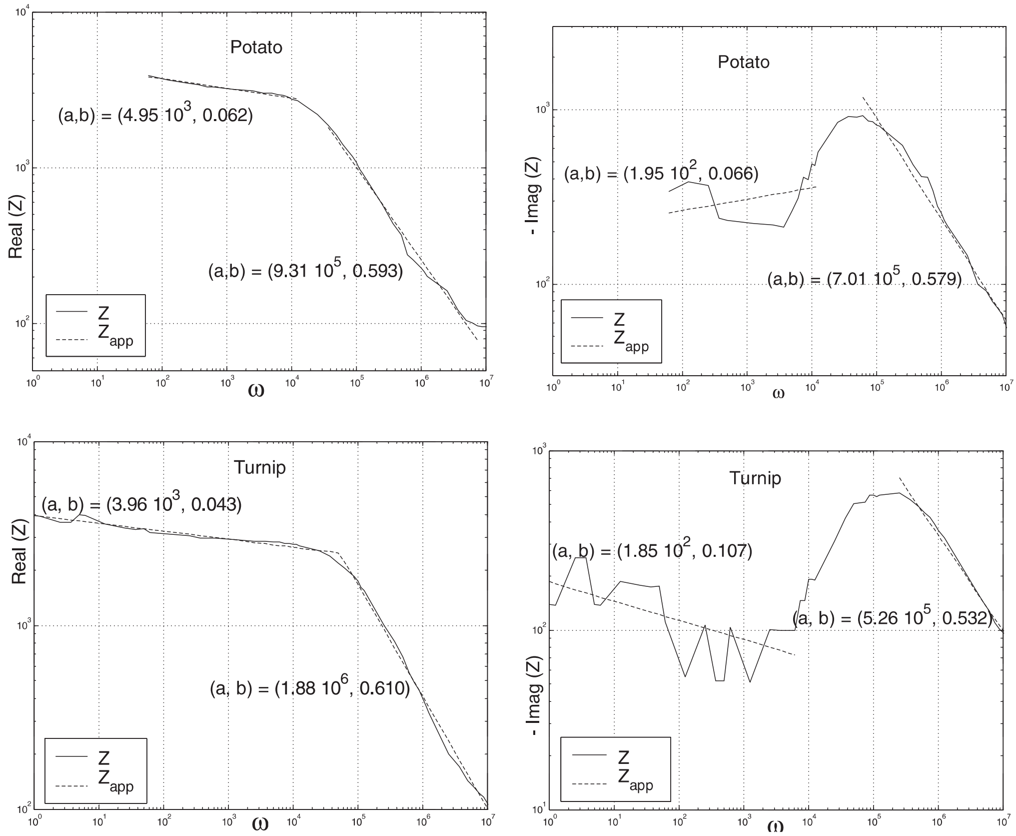

Figure 4. Diagrams of $\mathrm{Re}\{Z(j\omega)\}$ and $-\mathrm{Im}\{Z(j\omega)\}$ of the electrical impedance for several vegetables: Carrot (*Daucus Carota L.*) (with $R_a = 4.7\ \mathrm{k}\Omega$), garlic (*Allium sativum L.*) (with $R_a = 15.0\ \mathrm{k}\Omega$), onion (*Allium cepa L.*) (with $R_a = 2.7\ \mathrm{k}\Omega$), potato (*Solanum tuberosum*) (with $R_a = 15.0\ \mathrm{k}\Omega$), and turnip (*Brassica napobrassica*) (with $R_a = 2.2\ \mathrm{k}\Omega$). *(Continued)*

The results show that $Z(j\omega)$ has distinct characteristics for different frequency ranges. For low frequencies, the impedance is approximately constant, but for high frequencies, it is clearly of fractional order.

4. THE IMPEDANCE MODEL

In the previous section we presented asymptotic approximations (i.e., at low and high frequencies) because it is difficult to model $Z(j\omega)$ over the whole frequency range. In this section, we consider the circuit shown in Figure 6, which often adopted in the area of electrochemistry, where R_0 and R_1 are resistances and the *CPE* is given by expression (6).

The numerical values of R_0, R_1, C_F and α for the different impedances are shown in Table 3. The results reveal a very good fit for several vegetables and fruits. Figure 7 presents

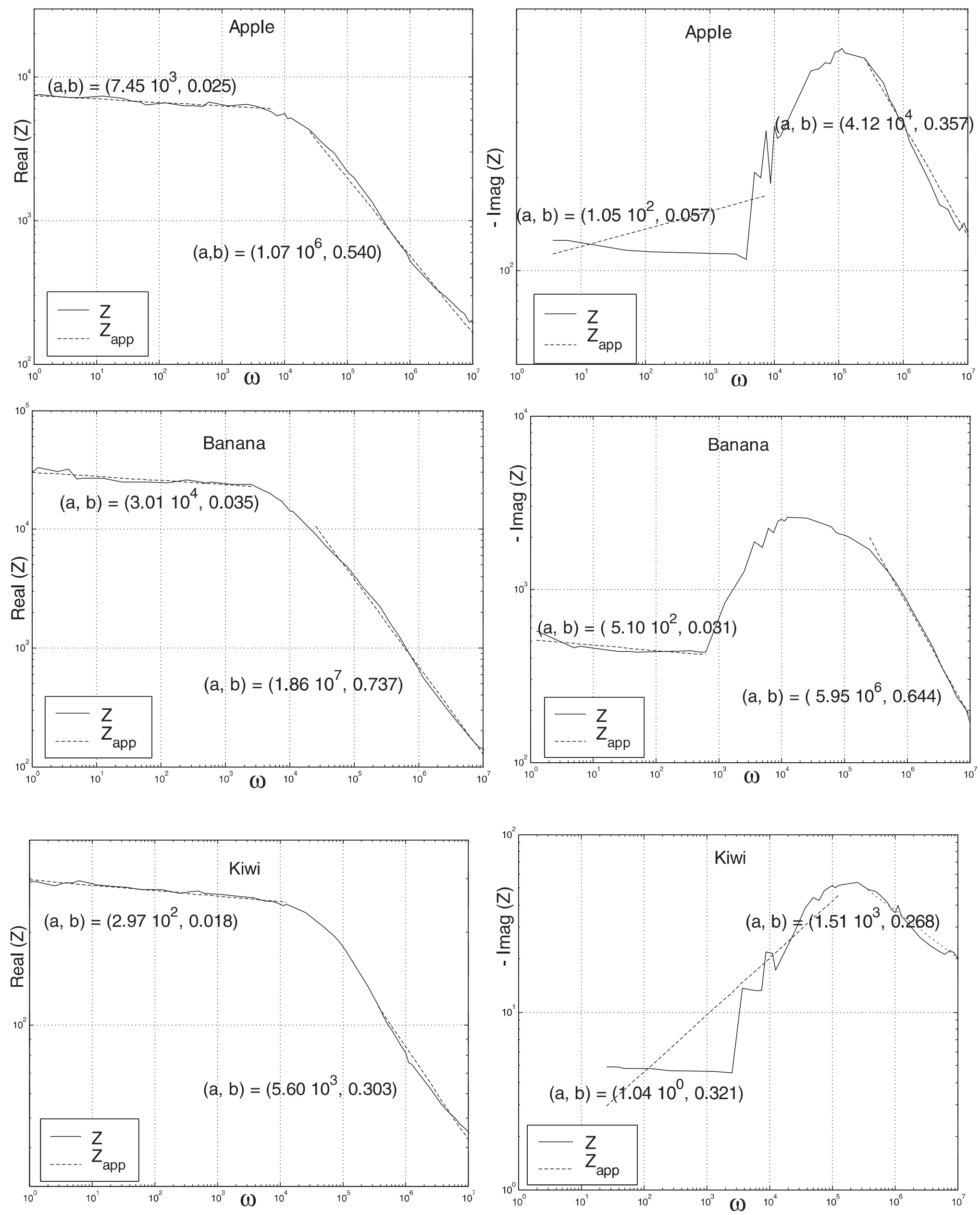

Figure 5. Diagrams of $\mathrm{Re}\{Z(j\omega)\}$ and $-\mathrm{Im}\{Z(j\omega)\}$ of the electrical impedance for several fruits: Apple (*Malus domestica*) (with $R_a = 1.0\ \mathrm{k}\Omega$), banana (*Musa ingens*) (with $R_a = 5.5\ \mathrm{k}\Omega$), kiwi (*Actinidia deliciosa*) (with $R_a = 750\ \Omega$), and lemon *citrus* × *limon* (with $R_a = 750\ \Omega$).

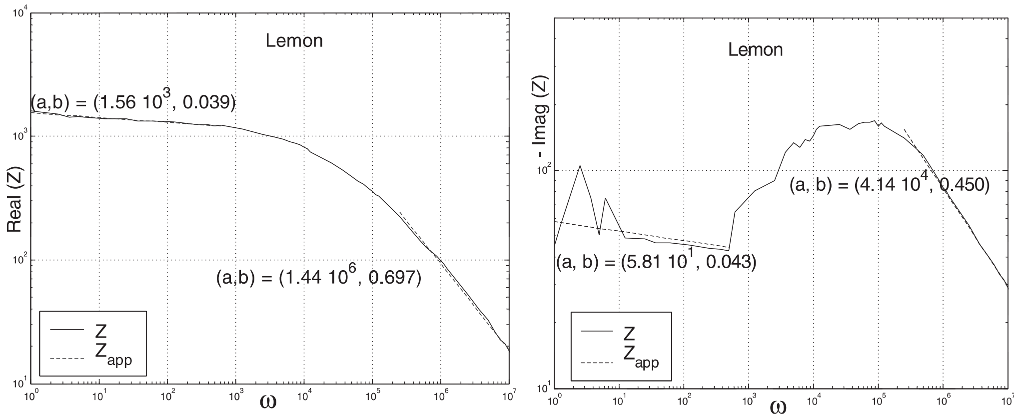

Figure 5. Diagrams of $\mathrm{Re}\{Z(j\omega)\}$ and $-\mathrm{Im}\{Z(j\omega)\}$ of the electrical impedance for several fruits: Apple (*Malus domestica*) (with $R_a = 1.0\ \mathrm{k}\Omega$), banana (*Musa ingens*) (with $R_a = 5.5\ \mathrm{k}\Omega$), kiwi (*Actinidia deliciosa*) (with $R_a = 750\ \Omega$), and lemon *citrus* × *limon* (with $R_a = 750\ \Omega$). *(Continued)*

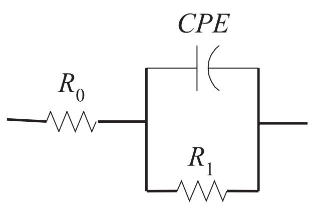

Figure 6. The Randles circuit.

Table 2. Characteristics of the vegetables and fruits.

Vegetable or Fruit (Species)	Weight (kg)	Length (m)	Width (m)
Carrot (*Daucus Carota L.*)	8.85×10^{-2}	1.55×10^{-1}	3.39×10^{-2}
Garlic (*Allium sativum L.*)	2.99×10^{-3}	1.38×10^{-2}	6.00×10^{-3}
Onion (*Allium cepa L.*)	8.33×10^{-2}	5.86×10^{-2}	5.77×10^{-2}
Potato (*Solanum tuberosum*)	1.24×10^{-1}	7.97×10^{-2}	5.99×10^{-2}
Pimento (*Capsicum annuum*)	1.30×10^{-1}	1.23×10^{-1}	8.20×10^{-2}
Tomato (*Lycopersicom esculentum*)	1.46×10^{-1}	5.57×10^{-2}	6.88×10^{-2}
Turnip (*Brassica napobrassica*)	7.90×10^{-2}	7.26×10^{-2}	5.43×10^{-2}
Apple (*Malus domestica*)	1.39×10^{-1}	6.36×10^{-2}	7.15×10^{-2}
Banana (*Musa ingens*)	1.11×10^{-1}	1.49×10^{-1}	3.42×10^{-2}
Kiwi (*Actinidia deliciosa*)	8.95×10^{-2}	6.52×10^{-2}	5.50×10^{-2}
Lemon (*Citrus × limon*)	1.66×10^{-1}	9.19×10^{-2}	6.58×10^{-2}
Orange (*Citrus sinensis*)	1.53×10^{-1}	6.69×10^{-2}	6.98×10^{-2}
Pear (*Pyrus communis*)	9.72×10^{-2}	6.51×10^{-2}	5.63×10^{-2}

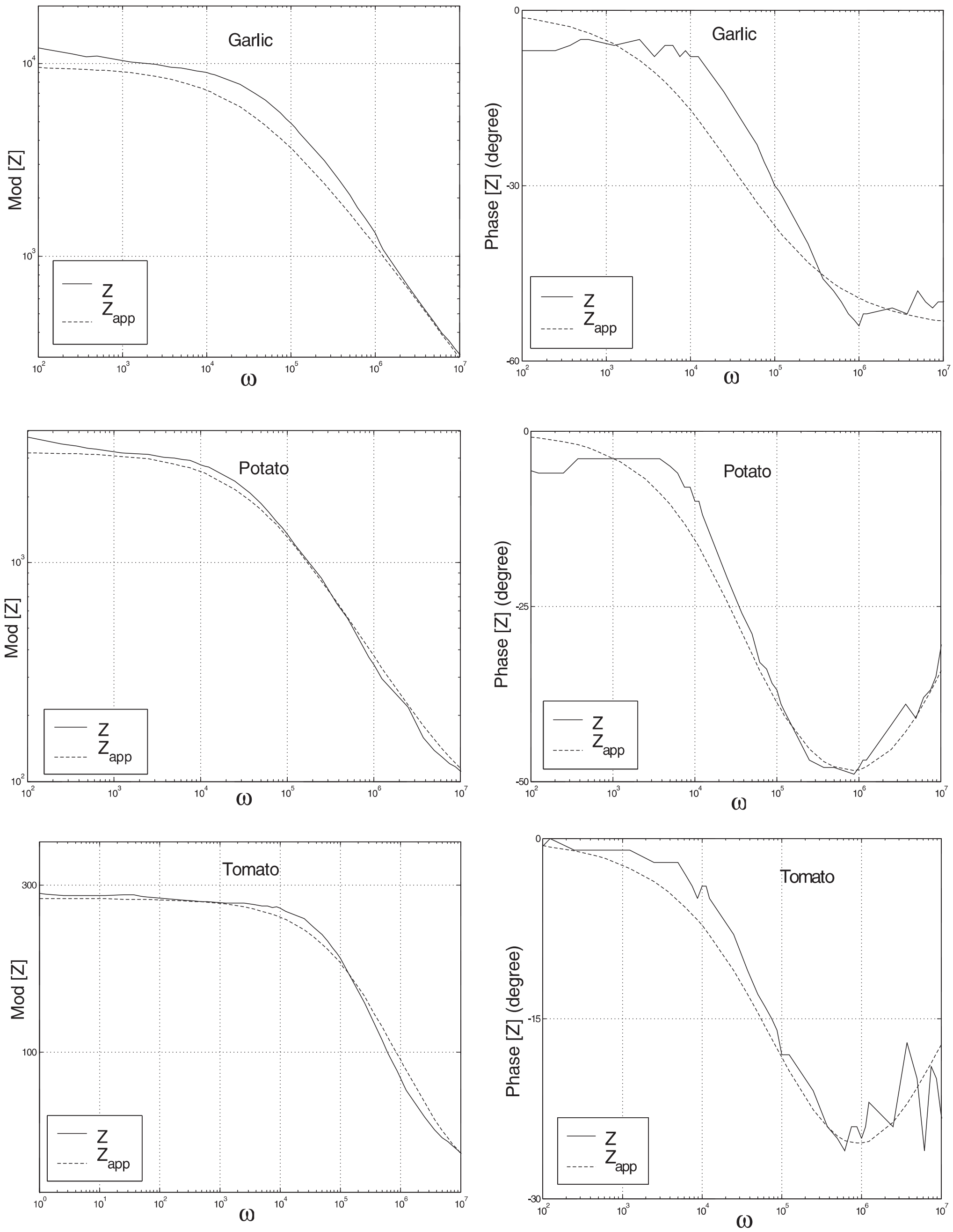

Figure 7. Amplitude and phase Bode diagrams of $Z(j\omega)$ for several vegetables and fruits: Garlic (*Allium sativum L.*), potato (*Solanum tuberosum*), tomato (*Lycopersicom esculentum*), kiwi (*Actinidia deliciosa*), and pear (*Pyrus communis*).

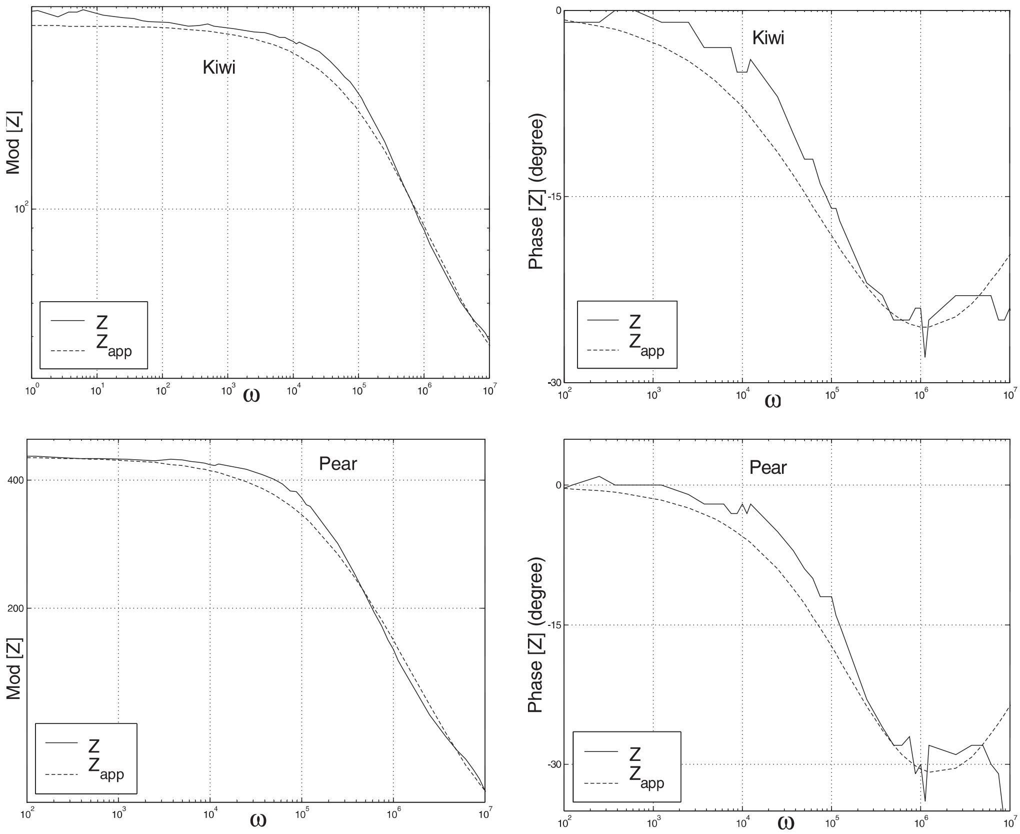

Figure 7. Amplitude and phase Bode diagrams of $Z(j\omega)$ for several vegetables and fruits: Garlic (*Allium sativum L.*), potato (*Solanum tuberosum*), tomato (*Lycopersicom esculentum*), kiwi (*Actinidia deliciosa*), and pear (*Pyrus communis*). *(Continued)*

Table 3. Values of the elements of the equivalent Randles circuit for various fruits and vegetables.

Vegetable/fruit	R_0 $[\Omega]$	R_1 $[\Omega]$	C_F $[m^{-2/\alpha} kg^{-1/\alpha} s^{(\alpha+3)/\alpha} A^{2/\alpha}]$	α
Garlic (*Allium sativum L.*)	1.00	9700	8.49×10^{-12}	0.609
Potato (*Solanum tuberosum*)	57.00	3150	1.67×10^{-10}	0.677
Tomato (*Lycopersicom esculentum*)	35.04	240.30	4.15×10^{-10}	0.565
Kiwi (*Actinidia deliciosa*)	28.04	242.00	2.33×10^{-10}	0.531
Pear (*Pyrus communis*)	44.04	409.00	2.51×10^{-10}	0.619

the amplitude and phase Bode diagrams for *Allium sativum L.* (garlic), *Solanum tuberosum* (potato), *Lycopersicom esculentum* (tomato), *Actinidia deliciosa* (kiwi fruit) and *Pyrus communis* (pear). Figure 8 contains the corresponding polar diagrams. It is clear that by adopting more complex circuits, we can achieve better approximations. However, models

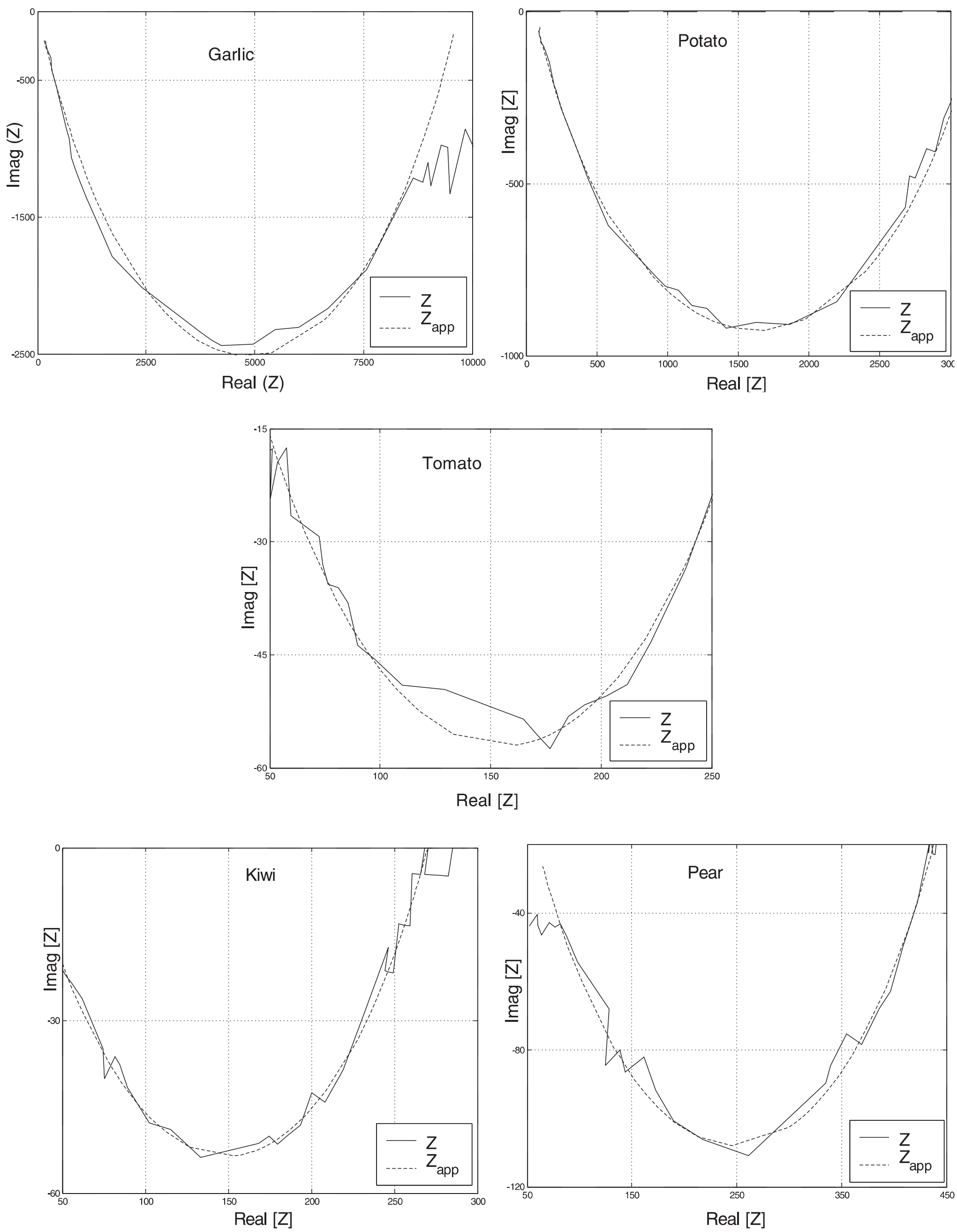

Figure 8. Polar diagrams of the impedance $Z(j\omega)$ for several vegetables and fruits: Garlic (*Allium sativum L.*), potato (*Solanum tuberosum*), tomato (*Lycopersicom esculentum*), kiwi (*Actinidia deliciosa*), and pear (*Pyrus communis*).

with a larger number of components make it more difficult to compare different cases, and to assign a physical meaning to each parameter.

Bearing these results in mind, we can conclude that the present study addresses the biological counterpart of technological fractional devices. In fact, recent research focused on the technological implementation of fractional order capacitances, including patents and available commercial products, open promising areas of application in electronics and control (Bohannan, 2002). This article follows an alternative strategy, studying natural living systems instead of artificial technological elements. Consequently, it points out new directions and constitutes a starting point towards the design of devices able to measure how mature a fruit or a vegetable is, or to give an estimative of its lifespan for storage purposes.

5. CONCLUSIONS

The idea of FC is not new and is, in fact, as old as its integer-order counterpart. The area of fractional calculus was, however, primarily the domain of mathematicians for some time, and had only theoretical foundations. Nowadays, the concept is employed in physics, engineering, biology, economics, and other scientific fields. In this work, we have applied the concepts of FC and electrical impedance to botanical elements. The fractional order behavior of these systems was studied, as well as its relation to the electrical impedance. The results reveal that all elements have different characteristics for low and high frequencies. In addition, it was verified that the impedance remains linear when the system conditions are modified. An equivalent circuit model was presented that is in accordance with the experiments developed for fruit and vegetables. The close fit of the experimental and analytical results indicates that the proposed model can be used to optimize the development of new methodologies, and in particular non-invasive electrode designs.

The impedances revealed fractional order characteristics at high frequencies, showing similarities with electrical fractional capacitors, also called fratances. The results demonstrate that fractional calculus is an important tool for describing physical phenomena, as adopting such alternative concepts can lead to perspectives that are not possible using classical approaches.

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