# Manipulability Analysis of Two-Arm Robotic Systems

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Abstract **%** Many tasks involving manipulation require cooperation between robots. Meanwhile, it is necessary to determinate the adequate values for the robot parameters to obtain a good performance. This paper the discusses several aspects related with manipulability co-operative of two robots when with handling objects different lengths and orientations. In this line of thought, a numerical tool is developed for the calculation and the graphical visualization of the manipulability measure.

#### I. INTRODUCTION

The choice of a robotic mechanism depends on the task or the type of work to be performed and, consequently, is determined by the position of the robots and by their dimensions and structure. In general, the selection is done through experience and intuition; therefore, it is important to formulate a quantitative measure of the manipulation capability of the robotic system, what can be useful in the robot control and in the trajectory planning. In this perspective Yoshikawa, proposed the concept of kinematic manipulability measure [1]. Several researchers tried to generalize the concept to dynamical manipulability [2, 4] and to a statistical evaluation of manipulation [5, 6]. Other related aspects such as the coordination of two robots handling objects, the collision avoidance and free path planning are also investigated in [7-11].

This paper analyses the kinematic manipulability measure and adapts the concept to a numerical/graphical technique. Bearing these facts in mind, this article is organized as follows. Section two, develops a numerical method for analyzing the manipulability of robotic systems. Based on the new algorithm, section three studies the performance of one-arm and two-arm systems. Finally, section four outlines the main conclusions.

# II. MANIPULABILITY OF ROBOTIC SYSTEMS

The manipulability measures the robot posture in the workspace from the viewpoint of object manipulation.

For one arm, Yoshikawa proposed, the manipulability index **m**given by:

$$\mathbf{m} = |\det[\mathbf{J}(\mathbf{q})]| \tag{1}$$

Where **J** is Jacobian of the robot kinematics. With this definition, for the RR robot it results  $\mathbf{m} = l_1 l_2 |sinq_2|$  where  $l_i$  and  $q_i$  (i = 1,2) are the length and position of link *i*,

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respectively. Based on this expression we can verify that the best posture for the RR robot occurs when  $q_2 = \pm 90^\circ$ . Besides, for a total length  $l_1+l_2 = L$ , the manipulability **m** has a maximum when  $l_1 = l_2$ .

For one robot the analytical development of **m** is straightforward; nevertheless, for two or more robots the definition of **m** is more complex. To overcome this problem we adopt a numerical approach inspired by the Monte Carlo method. In this perspective, we analyze both methods for a single robot, with the purpose of comparing the new numerical algorithm and the "classical" expression (1) and then we extend the concept for two robots working in cooperation.

The new method consists in generating a numerical sample of *n* points inside a sphere with radius **r**; in the joint space, and to map them to the operational space, in order to obtain a set of points corresponding to different ellipsoids. The size and shape of the ellipsoids determine the "amplification" between the joint space and the operational space. The amplification is related to eigenvalues of the Jacobian robot kinematics and corresponds to the area of the ellipsoid. The manipulability varies in the workspace, that is **m**  $\mathbf{m}(\mathbf{x}, y)$ ; therefore, we consider some indices to simplicity the study of the manipulability of several arms namely:

• The index **m** is defined as the maximum volume of **m** in all the possible workspace *W*.

$$\mathbf{m} = Max \left[ \mathbf{m}(x, y), \forall x, y \in W \right]$$

$$\tag{2}$$

• The index **m** is the average volume of **m** considering only the workspace W where **m**=0.

$$\mathbf{m} = Av \left[ \mathbf{m}(x, y), \forall x, y \in W: \mathbf{m}(x, y) \neq 0 \right]$$
(3)

• The index **m** represents the average volume of **m** in all the possible workspace *W*.

$$\mathbf{n} = Av \left[ \mathbf{n} (\mathbf{x}, y), \forall x, y \in W \right]$$

$$\tag{4}$$

# III. A NUMERICAL APPROCH FOR MANIPULABILITY

The following experiments adopt one and two robots with RR structure. In a first phase we consider one robot only, in order to compare the analytical and numerical methods. In a second phase, we consider two robots working in cooperation (figure 1), in order to determinate the manipulability of the total system and the system configuration that leads to a superior performance.



Fig. 1 - Two RR robots working in cooperation for the manipulation of an object with length *a* and orientation *c*.

## A. Manipulability of One Robot

Figure 2 shows the RR robot manipulability in the workspace obtained by the two alternative methods. As we can see, the numerical method presents a small error when compared with the analytical expression. Furthermore, the new algorithm has a low computational cost and it is easy to implement. Obviously, to decrease the numerical error it is necessary to increase the number n of samples, but the calculation time increases proportionally.



Fig. 2 - Manipulability **m** f one RR robot with  $l_1 = 1$  and  $l_2 = 0.8$  obtained by the: a) analytical method, b) numerical algorithm for a sample of n = 1000 points.

#### B. Manipulability of Two Robots Working in Cooperation

In this sub-section, we consider two robots working in cooperation. In this way, we start with several experiments to obtain the manipulability of the two arms in its workspace for a = 0 (small objects) and  $b \in [0, 2(l_1 + l_2)]$ .

Given the kinematic redundancy of the two-arm system, for each grasping point we consider that, the left and the right arms define, alternatively, the hand position. Moreover, we establish a grid of *m* points in the workspace and, for each of these points, we generate a sample of npoints, in the interior of a sphere with a radius  $\boldsymbol{r}$  in the joint space.

Figure 3 shows the manipulability in the workspace of two robots working in cooperation for  $b \in [0, 4]$  and the cases  $A = \{l_1 = 0.5, l_2 = 1.5\}, B = \{l_1 = 1.5, l_2 = 0.5\}, C = \{l_1 = 1, l_2 = 1\}.$ 

The chart shows m as function of the distance b between the arm elbows and reveals that we obtain larger values for  $b \approx 0$  because the workspace is maximum in that case, while the best case occurs for  $l_1 = l_2$ .



Fig. 3 - Manipulability **m** for the considered basis for  $A = \{l_1 = 0.5, \dots, l_{n-1}\}$  $l_2 = 1.5$ ,  $B = \{l_1 = 1.5, l_2 = 0.5\}$ ,  $C = \{l_1 = 1, l_2 = 1\}$ .

Therefore, we can say that, when manipulating small objects, the distance between the arms should be b = 0, for arms having  $l_1 = l_2 = L/2$ . Nevertheless, studying the human body we see that it presents  $l_1 = l_2$  and L/2 < b < 3L/2Therefore, the parameters a and c must influence the manipulability and this hypothesis must be investigated. In this line of thought, we consider for the first study two identical RR robots with  $l_1 = l_2$  cooperating in the manipulation of objects with non-zero dimension  $a = \{0,2,4\}$  while varying namely  $b \hat{I}[0, 2(l_1 + l_2) + a]$  and an

object orientation  $c \in [-180^\circ, +180^\circ]$ .

Figures 4 - 6 show the indices m, m and m versus the parameters a, b, and c. This numerical experiment considers a grid of m = 1000 points and, for each of these points, a sample of n = 1000 points, inside a sphere with a radius of  $\mathbf{r} = 0.1$  rad in the joint space.

In the second study we consider robot 1 larger than the robot 2 and then we repeat the experiments in order to compare the manipulability indices. Therefore, we establish robot 1 with  $l_1 = l_2 = 1.3L$  and robot 2 with  $l_1 = l_2 = 0.7L.$ 

The results are presented in figures 7-9 and we conclude that for the case of different robots we have a manipulability reduction and the appearance of a "hole" in the left part of the chart. Figures 10-11 show the relationship between the length *a* and the distance between two arms b. We can observe that, in both cases, we get a maximum manipulability for a = b

## **IV. CONCLUSIONS**

This paper developed a study of the manipulability of two robots, with RR structure, working in cooperation. In this perspective, it was introduced a numerical tool for the analysis of the kinematic manipulability of one or more robots in the workspace. Based on the new algorithm, three distinct indices were evaluated in order to characterize the system manipulability. It was possible to compare several situations, such as different sizes and orientations of the object and distinct lengths between the two arms.

We conclude that the maximum manipulability occurs for  $l_1=l_2$  and c=0. Therefore, it is interesting to investigate, in more detail, the influence of the parameters a and b when c = 0. This analysis revealed that the best manipulability occurs when a = b for c = 0.



Fig. 4 – The maximum manipulability **m** versus b and c for object lengths  $a = \{0, 2, 4\}$ , with  $m = 1000, n = 1000, \mathbf{r} = 0.1$  rad, Robot 1= Robot 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ .



Fig. 5 – Two arm average volume of the manipulability **m** versus b and c for object lengths  $a = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, Robot 1 = Robot 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ .



Fig. 6 – Two-arm average manipulability for the considered basis **m** versus b and c for object lengths  $a = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, Robot 1 = Robot 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ 



Fig. 7 – The maximum manipulability **m** versus *b* and *c* for object lengths  $a = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, Robot 1:  $\{l_1 = l_2 = 1.3 \text{ m}\}$ , Robot 2:  $\{l_1 = l_2 = 0.7 \text{ m}\}$ .



Fig. 8 – Two arm average volume of the manipulability **m** versus b and c for object lengths  $a = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, Robot 1:  $\{l_1 = l_2 = 1.3 \text{ m}\}$ , Robot 2:  $\{l_1 = l_2 = 0.7 \text{ m}\}$ .



Fig. 9 – Two-arm average manipulability for the considered basis **m** versus b and c for object lengths  $a = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, Robot 1:  $\{l_1 = l_2 = 1.3\}$ , Robot 2:  $\{l_1 = l_2 = 0.7$  m}.



Fig. 10 – The indices of manipulability **m** m and **m** versus a and b for an object orientation c = 0, with  $a \in [0, 10]$  and  $b \in [0, 5]$ , m = 1000, n = 1000, r = 0.1 rad, Robot 1= Robot 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ .



Fig. 11 – The indices of manipulability **m** m and **m** versus a and b for an object orientation c = 0, with  $a \in [0, 10]$  and  $b \in [0, 5]$ , m = 1000, n = 1000,  $\mathbf{r} = 0.1$  rad, Robot 1:  $\{l_1 = l_2 = 1.3 \text{ m}\}$ , Robot 2:  $\{l_1 = l_2 = 0.7 \text{ m}\}$ .

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