# Manipulation Analysis of Two Cooperating Arms

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**Abstract:** Many tasks involving manipulation require cooperation between robots. Meanwhile, it is necessary to determinate the adequate values for the robot parameters to obtain a good performance. This paper discusses several aspects related with the manipulability and the workspace of two co-operative robots when handling objects with different lengths and orientations.

### 1. Introduction

The choice of a robotic mechanism depends on the task or the type of work to be performed and, consequently, is determined by the position of the robots and by their dimensions and structure. In general, the selection is done through experience and intuition; nevertheless, it is important to measure the manipulation capability of the robotic system [1], what can be useful in the robot control and in the task planning. In this perspective it was proposed the concept of kinematic manipulability measure [2] and its generalization dynamical manipulability [3, 4] or, alternatively, to a statistical evaluation of manipulation [5, 6]. Moreover, other related aspects such as the coordination of two robots handling objects, collision avoidance and free path planning have been also investigated [7-12] but still require further study.

This paper analyses the manipulation measure and capability of two-arm systems using a numerical/graphical technique. Bearing these facts in mind, this article is organized as follows. Section two, develops a numerical method for analyzing the manipulability of robotic systems. Based on the new algorithm, section three studies the performance of two-arm systems. Section four evaluates the workspace of two-arm systems and, finally, section five outlines the main conclusions.

## 2. Manipulability of Robotic Systems

The manipulability measures the robot efficiency in the workspace from the viewpoint of object manipulation. For one arm the kinematic manipulability **m** is defined as:

$$\boldsymbol{m} = |\det[\boldsymbol{J}(\mathbf{q}) \, \boldsymbol{J}^{T}(\mathbf{q})]|^{1/2} \tag{1}$$

where J is Jacobian of the robot kinematics. With this formulation, for example, for the *RR* robot it yields  $\mathbf{m} = l_1 l_2 |\sin q_2|$ , where  $l_i$  and  $q_i$  (i = 1,2) are the length and position of link *i*, respectively. Based on this expression we can verify that the best posture for the *RR* robot

occurs when  $q_2 = \pm 90^{\circ}$  and, for a total length  $l_1 + l_2 = L$ , **m** has a maximum when  $l_1 = l_2$ .

For one robot the analytical development of **m** is straightforward; however, for two or more robots the definition of **m** is more complex. To overcome this problem we adopt a numerical approach inspired by the Monte Carlo method. In this perspective, we analyze both methods for a single robot, with the purpose of comparing the numerical algorithm and the classical expression (1) and then we extend the concept for two robots working in cooperation.

For a k-dof manipulating structure, the new method [13] consists in generating a numerical random sample of n points inside a sphere with radius  $\mathbf{r}$  (Fig. 1), in the joint space with center at  $(q_{1c},...,q_{kc})$ , and to map them through the direct kinematics to the operational space, in order to obtain a set of points corresponding to an ellipsoid with center at  $(x_c, y_c)$ . The size and shape of each ellipsoid determine the "amplification" between the joint space and the operational space. The amplification is related to eigenvalues of the Jacobian robot kinematics and corresponds to the area of the ellipsoid. The manipulability varies in the workspace W and, in order to analyze its texture, we establish a grid of m

points for the centers of the sphere  $\rightarrow$  ellipsoid transformation. Therefore, we consider some subindices to simplify the study of the manipulability of several arms. By other words, to condense the information we define:

• The sub-index **m** as the maximum value of **m** in all the possible workspace *W*.

$$\mathbf{m} = Max \left[ \mathbf{m}(x, y), \forall (x, y) \in W \right]$$
(2)

• The sub-index  $\mathbf{m}$  as the average volume of  $\mathbf{m}$  considering only the workspace W where  $\mathbf{m} \neq 0$ .

$$\mathbf{m} = Av \left[ \mathbf{m}(x, y), \forall (x, y) \in W: \mathbf{m}(x, y) \neq 0 \right]$$
(3)

• The index **m** as average volume of **m** in all the possible workspace *W*.

$$\mathbf{m} = Av \left[ \mathbf{m}(x, y), \forall (x, y) \in W \right]$$
(4)

#### 3. A Numerical Approach for Manipulability

The following experiments adopt one and two robots with RR structure. In a first phase we consider a single robot, in order to compare the analytical and numerical methods. In a second phase, we consider two robots working in cooperation (figure 2), in order to determinate the manipulability of the total system and the system configuration that leads to a superior performance.



Fig. 1 - Manipulability analysis through the numerical method.



Fig. 2 - Two *RR* robots working in cooperation for the manipulation of an object with length  $l_0$ , orientation **a**<sub>0</sub> and distance  $l_b$  between the shoulders.

## 3.1. Manipulability of One Robot

Figures 3 shows the manipulability for one *RR* robot in the workspace obtained by the two alternative methods.



Fig. 3 - Manipulability **m**of one *RR* robot with  $l_1 = 1$  m and  $l_2 = 0.8$  m obtained by the: a) analytical method, b) numerical algorithm for a sample of n = 1000 points and a grid of m = 1000 points.

As we can see, the numerical method presents a small error when compared with the analytical expression. Furthermore, the new algorithm has a low computational cost and it is easy to implement. Obviously, to decrease the numerical error it is necessary to increase the number n of samples, but the calculation time increases proportionally.

#### 3.2. Manipulability of Two Cooperating Robots

In this sub-section, we consider two robots working in cooperation. In this way, we start with several experiments to obtain the manipulability of the two arms in its workspace for  $l_o = 0$  (small objects) and  $l_b \in [0, 2(l_1 + l_2)]$ .

Given the kinematic redundancy of the two-arm system, for each grasping point we consider that, the left and the right arms define, alternatively, the hand position.

Figure 4 shows the manipulability  $\mathbf{m}$  in the workspace of two *RR* robots working in cooperation for  $l_b \in [0, 4[$  and the cases  $A = \{l_1 = 0.5, l_2 = 1.5\}, B = \{l_1 = 1.5, l_2 = 0.5\}, C = \{l_1 = 1, l_2 = 1\}$ . The chart shows **m** as function of the distance  $l_b$  between the arm elbows and reveals that we obtain larger values for  $l_b \approx 0$  because the workspace is maximum in that case, while the best case occurs for  $l_1 = l_2$ 



Fig. 4 - Manipulability **m** for  $A = \{l_1 = 0.5, l_2 = 1.5\}, B = \{l_1 = 1.5, l_2 = 0.5\}, C = \{l_1 = 1, l_2 = 1\}.$ 

Therefore, we can say that, when manipulating small objects, the distance between the arms should be  $l_b = 0$ , for arms having  $l_1 = l_2 = 1/2L$ , with L = 2 m. Nevertheless, studying the human body we see that it presents  $l_1 = l_2$  and  $1/2L < l_b < 3/2L$ . Therefore, the parameters  $l_o$  and **a** must influence the manipulability and this hypothesis must be investigated. In this line of thought, we consider for the first study two identical *RR* robots with  $l_1 = l_2$  cooperating in the manipulation of objects with dimension  $l_o = \{0,2,4\}$  while varying  $l_b \in [0, 2(l_1+l_2) + l_o]$  and **a**  $\in [-180^\circ, +180^\circ]$ .

Figures 5 – 7 show the indices  $m_1$ ,  $m_2$  and  $m_2$  versus the parameters  $l_o$ ,  $l_b$  and  $a_b$  for two RR robots. This numerical experiment considers a grid of m = 1000 points and, for each of these points, a sample of n = 1000 points, inside a sphere with a radius of r = 0.1 rad in the joint space.

Figure 8 shows the relationship between the length  $l_o$  and the distance between the arms  $l_b$ . We can observe that we get a maximum manipulability for  $l_o = l_b$ .



Fig. 5 – Two arm maximum manipulability **m** versus  $l_b$  and **a**, for object lengths  $l_o = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, *RR*-Robots 1 and 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ .



Fig. 6 – Two arm average volume of the manipulability  $\mathbf{m}$  versus  $l_b$  and  $\mathbf{a}_b$  for object lengths  $l_o = \{0, 2, 4\}$ , with m = 1000, n = 1000,  $\mathbf{r} = 0.1$  rad, *RR*-Robots 1 and 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ .



Fig. 7 – Two average manipulability for the considered basis **m** versus  $l_b$  and **a** for object lengths  $l_o = \{0, 2, 4\}$ , with m = 1000, n = 1000, r = 0.1 rad, *RR*-Robots 1 and 2:  $\{l_1 = l_2 = 1 \text{ m}\}$ .



Fig. 8 – The sub-indices of manipulability  $\mathbf{m}_{i}$   $\mathbf{m}_{j}$  and  $\mathbf{m}_{j}$  versus  $l_{o} \in [0, 10]$  and  $l_{b} \in [0, 5]$ , for an object orientation  $\mathbf{a}_{b} = 0$ , m = 1000, n = 1000,  $\mathbf{r} = 0.1$  rad, RR-Robots 1 and 2:  $\{l_{1} = l_{2} = 1 \text{ m}\}$ .

#### 4. Workspace Analysis

In this section we analyze another important aspect of robot co-operation, namely the system workspace and its properties. In this line of thought, we compare the workspace for experiments having distinct robot joint-angle limits ( $q_{imax}$  and  $q_{imin}$ , i = 1, 2) and several distances  $l_b$  between the robots.

For large values of  $l_b$ , required in the body for handling large objects, the workspace is just a small 'ellipse', without holes, because the robots are so separated apart that it is impossible to have contacts between them and/or the object. Nevertheless, this situation is far from the case of the human body and, therefore, in our experiments we will consider the case of  $l_b \le l_1+l_2+l_0$ ,  $l_o = l_b = 1$  and  $\mathbf{a}_b = 0$ .

For comparison, the first experiment depicts the workspace of two RR arms with no limitations on the robot joints of Table 1 while the second and third experiments consider the joints limitations of Tables 2 and 3, respectively. Figures 9 to 11 show the corresponding workspace and manipulability.



Robot	$q_{1min}$ (rad)	$q_{1max}$ (rad)	$q_{2min}$ (rad)	$q_{2max}$ (rad)
1	-π	π	-π	π
2	$-\pi$	0	0	π



Fig. 9 – Two *RR*-arm manipulability **m** and workspace for  $a_b = 0$ ,  $l_o = l_b = 1$ , m = 1000, n = 1000, *RR*-Robots 1: and 2 { $l_1 = l_2 = 1$  m} and the joint angle limitations of Table 1.

Figure 9 shows an "hole" in the lower side of the workspace that is due to the chock between the object and the two arms. Moreover, the chart reveals that the region of the maximum manipulability in the operational

workspace is in the same place as the one shown in the single arm case. On the other hand, Figures 10-11 show that the larger diminishing of the workspace is due, obviously, to the joint angle limitations.

Table 2 – Two *RR* arm system with light joint limitations

Robot	$q_{1min}$ (rad)	$q_{1max}$ (rad)	$q_{2min}$ (rad)	$q_{2max}$ (rad)
1	0	π	0	π
2	$-\pi$	0	0	π



Fig. 10 – Two *RR*-arm manipulability **m** and workspace for  $\mathbf{a}_b = 0$ ,  $l_o = l_b = 1$ , m = 1000, n = 1000, *RR*-Robot 1 and 2:  $\{l_1 = l_2 = 1 \text{ m}\}$  and the joint angle limitations of Table 2.

Table 3- Two RR arm system with strong joint limitations

Robot	$q_{1min}$ (rad)	$q_{1max}$ (rad)	$q_{2min}$ (rad)	$q_{2max}$ (rad)
1	0	π	0	π
2	$-8/9 \pi$	0	0	$8/9 \pi$



Fig. 11 – Two *RR*-arm manipulability **m** and workspace for  $\mathbf{a}_b = 0$ ,  $l_o = l_b = 1$ , m = 1000, n = 1000, *RR*-Robots 1: and 2:  $\{l_1 = l_2 = 1 \text{ m}\}$  and the joint angle limitations of Table 3.

Note that the third experience considers joints limitations that resemble the human arm case. However, if we study the human body and the manipulation through two arms we observe that it is not 'prepared' to manipulate very large objects. In fact, our arms have a good working region near the body and, as we shall see, superior than the case of two *RR* mechanical arms. Consequently, to improve the workspace properties the system must have cooperative robots with kinematic redundancy [14].

In this perspective we adopt two *RRR* co-operating arms so that the total length of each arm remains L = 2 m for the two cases *i*)  $l_1 = l_2 = l_3 = 2/3$  m and *ii*)  $l_1 = 5/6$  m,  $l_2 = 2/3$  m and  $l_3 = 1/2$  m. Moreover, due to the redundancy, there is an infinite number of arm configurations (and, consequently, of values of  $\mu$ ) for each manipulating point (*x*,*y*). Therefore we consider the average manipulability  $\mu_{A\nu}$  to captures all possibilities.

Figures 12-13 show  $\mu_{Av}$  for robots without joint angle limitations (Table 4) when considering the case of  $l_o = l_b = 1$  m and  $a_b = 0$ .



Fig. 12 – Two *RRR*-arm  $\mu_{A\nu}$  and workspace for  $\mathbf{a}_{b} = 0$ ,  $l_{o} = l_{b} = 1$ , *RRR*-Robots 1 and 2: { $l_{1} = l_{2} = l_{3} = 2/3$  m} and the joint angle limitations of Table 4.



Fig. 13 – Two *RRR*-arm  $\mu_{A\nu}$  and workspace for  $\mathbf{a}_b = 0$ ,  $l_o = l_b = 1$ , *RRR*-Robot 1 and 2: { $l_1 = 5/6$  m,  $l_2 = 2/3$  m,  $l_3 = 1/2$  m} and the joint angle limitations of Table 4.

Figures 14-15 show the workspace and the manipulability with joint angle limitations (Table 5) when considering the case of  $l_o = l_b = 1$  m and  $\mathbf{a}_b = 0$ .



Fig. 14 – Two *RRR*-arm  $\mu_{A\nu}$  and workspace for  $\mathbf{a}_b = 0$ ,  $l_o = l_b = 1$ , *RRR*-Robots 1 and 2:  $\{l_1 = l_2 = l_3 = 2/3 \text{ m}\}$  and the joint angle limitations of Table 5.



Fig. 15 – Two *RRR*-arm  $\mu_{A\nu}$  and workspace for  $\mathbf{a}_b = 0$ ,  $l_o = l_b = 1$ , *RRR*-Robot 1 and 2: { $l_1 = 5/6$  m,  $l_2 = 2/3$  m,  $l_3 = 1/2$  m} and the joint angle limitations of Table 5.

Robot	$q_{1min}$ (rad)	$q_{1max}$ (rad)	$q_{2min}$ (rad)	$q_{2max}$ (rad)	$q_{3min}$ (rad)	$q_{3ax}$ (rad)
1	$-\pi$	π	$-\pi$	π	-π/2	π/2
2	$-\pi$	π	$-\pi$	π	-π/2	π/2

Table 5 – Two *RRR* arm system with joint limitations

Robot	$q_{1min}$ (rad)	$q_{1max}$ (rad)	$q_{2min}$ (rad)	$q_{2max}$ (rad)	$q_{3min}$ (rad)	$q_{3ax}$ (rad)
1	0	π	$-8/9 \pi$	8/9 π	<b>-π</b> /2	π/2
2	0	π	$-8/9 \pi$	8/9 π	$-\pi/2$	π/2

When considering the joint angle limitations (Table 5) it is straightforward that the kinematic redundancy has improved largely the workspace area. Furthermore, the workspace of case *i*) seems inferior to the case *ii*), namely from the point of view of human manipulation. In fact, the reduction of the work area in the back part of the body is less important than the elimination of the 'hole' revealed in *ii*), near the center-frontal part of the body.

## 5. Conclusions

This paper developed a study of the manipulation capability of two robots working in cooperation. In this perspective, a numerical tool was introduced for the analysis of the kinematic manipulability of multiple robots in the workspace. Based on the new algorithm several indices were evaluated in order to characterize the system manipulability. It was possible to compare distinct situations, such as different sizes and orientations of the object and distinct lengths between the two arms. Moreover, the kinematic redundancy and joint angle limitations were also investigated, in order to compare the numerical results and the characteristics of the human beings.

#### References

 Y. C. Tsai and A.H Soni, "Accessible Region and Synthesis of Robot Arms," ASME J. Mech. Design, vol. 103, pp. 803-811, Oct. 1981.

- [2] T. Yoshikawa, "Manipulability of Robotic Mechanisms," The Int. J. Robotics Research, vol. 4, pp. 3-9, Summer, 1985.
- [3] H. Asada, "A Geometrical Representation of Manipulator Dynamics and its Application to Arm Design," ASME J. Dynamic Syst., Meas., Contr., vol. 105, pp. 131-142, 1983.
- [4] V. Scheinman and B. Roth, "On the Optimal Selection and Placement of Manipulators," RoManSy'84: The Fifth CISM-IFToMM Symp., Udine, Italy, 1984.
- [5] A. M. Galhano, J. M. de Carvalho and J. A. T. Machado, "The Statistical Study of Robot Manipulators," IEEE Int. Symp. on Intelligent Control, Philadelphia, Pennsylvania, USA, 1990.
- [6] J. A. T. Machado and A. M. Galhano, "A Statistical and Harmonic Model for Robot Manipulators," IEEE Int. Conf. on Robotics and Automation, Albuquerque, New Mexico, USA, 1997.
- [7] Y. P. S. Chien, Q. L. Xue, Y. Chen, "C- Subspace Model of Tightly Coordinated Two-Planar Robots", International Journal of Robotics and Automation, vol. 8, N°1, 1989.
- [8] Y. Nakamura, K. Nagai, T. Yoshikawa, "Dynamics and Stability in Coordination of Multiple Robotic Mechanisms," International Journal of Robotics Research, vol. 8, pp. 44-61, 1989.
- [9] B. Cao, G. I. Dodds, G. W. Irwin, "An Approach to Time Optimal, Smooth and Collision-Free Path Planning in a Two Robot Arm Environment" International Journal of Robotics, vol. 14, pp 61-70, 1996.
- [10] T. J. Tarn, A. K. Bejczy, P. K. De, "Analysis of the Dynamic Ability of Two Robot Arms in Object Handling", Advanced Robotics, vol. 10, n. 3, pp. 301-315, 1996.
- [11] C. Y. Kim, Y. S. Yoon, "Task Space Dynamic Analysis for Multi-Arm Robot Using Isotropic Velocity and Acceleration Radii", Int. J. of Robotics, vol. 15, pp. 319-329, 1997.
- [12] N. M. Fonseca Ferreira, J. A. Tenreiro Machado, "Manipulability Analysis of Two-Arm Robotic Systems", 4th IEEE International Conference on Intelligent Engineering Systems, Portoroz, Slovenia, Sept. 2000.
- [13] A. K. Bejczy and T. Jonhg Tarm, "Redundancy in Robotics Connected Robots Arms as Redundant Systems", 4th IEEE International Conference on Intelligent Engineering Systems, Portoroz, Slovenia, Sept. 2000.
- [14] Fernando B. M. Duarte, J. A. Tenreiro Machado, "Chaos Dynamics in the Trajectory Control of Redundant Manipulators", IEEE Int. Conference on Robotics and Automation, San Francisco, USA, April 2000.