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FRACTIONAL-ORDER DYNAMICS IN FREEWAY TRAFFIC

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Abstract: This paper presents the *Simulator of Intelligent Transportation Systems* (*SITS*). The *SITS* is based on a microscopic simulation approach to reproduce real traffic conditions, both in urban and non-urban networks, and considers different types of vehicles, drivers and roads. A dynamical analysis of several traffic phenomena is then addressed. The results of using classical system theory tools point out that it is possible to study traffic systems taking advantage on the knowledge gathered with automatic control algorithms. In this line of thought, it is also presented a new modelling formalism based on the embedding of statistics and Fourier transform.

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1. Introduction

Nowadays we have a saturation of the transportation infrastructures due to the growing number of vehicles over the last five decades. This situation affects substantially our lives particularly in the urban areas, resulting in traffic congestion, accidents, transportation delays and larger vehicle pollution emissions. The difficulties concerned with this subject motivated the research community to center their attention in the area of ITS (Intelligent Transportation Systems). ITS applies advanced communication, information and electronics technology to solve transportation problems such as, traffic congestion, safety, transport efficiency and environmental conservation. Therefore, we can say that the purpose of ITS is to take advantage of the appropriate technologies to create "more intelligent" roads, vehicles and users [1]. Computer simulation has become a common tool in the evaluation and development of ITS. The advantages of this tool are obvious. The simulation models can satisfy a wide range of requirements, such as: evaluating alternative treatments, testing new designs, training personal and analyzing safety aspects. Bearing these facts in mind, this paper is organized as follows. Section 2 discusses the modelling and simulation of ITS. Section 3 describes briefly the microsimulation model *SITS*. Section 4 presents simulation results related with the dynamic behaviour of a traffic system. Finally, Section 5 presents some conclusions and outlines the perspectives towards future research.

2. Modelling and Simulation

This section presents an overview of the development of simulation models in road traffic planning and research, which is considered as the most prevalent in the transportation community. However, it should be noted that there are many other simulation models available for use in aviation, railroad and maritime transportation. The first research work on this subject was published in 1955 at the University of California, by D.L. Gerlough under the title "Simulation of freeway traffic on a general-purpose discrete variable computer". The car-following analysis based on GM models, is one of the oldest and most well known cases of the use of simulation in theoretical research. In these models, the movement of each vehicle in the platoon under analysis is governed by a differential equation [4]. After almost 40 years from the first trials, car-following is still under active analysis, being one of the basic questions of traffic flow theory. In the recent years simulation models have been developed to support the analysis in almost all the areas of ITS namely in Advanced Traveller Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS). The traffic simulation models can be classified according to various criteria, namely, the scale of independent variables, the representation of the processes and levels of detail [5]. The classification according to the level of detail with which the traffic system is represented by the model can be divided in microscopic, mesoscopic and macroscopic. The microscopic simulation model describes both, the space-time behaviour of the system entities (i.e., vehicles and drivers) as well as their interactions at a high level of detail (individually). The mesoscopic model represents most entities at a high level of detail, but describes their activities and interaction at a lower level of detail. The macroscopic model represents entities and describes their activities and interactions at a low level of detail. These models describe traffic at a high level of aggregation as a flow without distinguishing its constituent parts. Presently, most traffic system simulation applications are microscopic in nature and based on the simulation of vehicle-vehicle interactions [7]. Microscopic traffic simulators are simulation tools that emulate realistically the flow of vehicles on a road network. Micro-simulation is used for evaluation prior to, or in parallel with, on-street operation. The main modelling components of a microscopic traffic simulation model are: an accurate representation of the road network geometry, a detailed modelling of individual vehicles behaviour and an explicit reproduction of traffic control plans. With these components it is possible to deal with ITS systems, like adaptive traffic control systems, automatic incident detection systems, dynamic vehicle guidance systems and advanced traffic management systems. The recent evolution of the microscopic simulators has taken advantages of the state-of-the-art in the development of object-oriented simulators and graphical user interfaces. There are a considerable number of developed microscopic simulation models. The *SMARTEST* project identified 58 of these models of which the most important are listed on Table 1.

The model type "other" have been designed with specific objectives like modelling of the tactical level of driving and testing of intelligent vehicle algorithms. They provide a detailed roadway environment for a simulated robot-driving vehicle, to evaluate the safety and comfort conditions of a cars line on a single lane or to simulate strategies [9]. An important issue is the modelling and simulation of the driver steering behaviour. Due to the development of vehicles incorporating new technological devices, a deeper knowledge about the interaction between the vehicle and the driver becomes of great usefulness for the vehicle design. Usually, in simulation programs, the drivers are programmed to avoid collisions, so they do not exist. Although some trials for analysis of conflict situations have been made, a general approach to this problem is still missing. Traffic safety simulation is sometimes classified as nanosimulation, belonging to the field of human centred simulation where the perception-reaction system of drivers and all its characteristics are described [6].

3. The SITS Simulation Package

SITS is a software tool based on a microscopic simulation approach, which reproduces real traffic conditions in an urban or non-urban network. The program provides a detailed modelling of the traffic network, distinguishing between different types of vehicles and drivers and considering a wide range of network geometries.

SITS models each vehicle as a separate entity in the network according to the state diagram showing in Figure 1. Therefore, are defined five states (1-acceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision) that represent the possible vehicle states in a traffic systems.

In this modelling structure, so called State-Oriented Modelling (SOM), every single vehicle in the network has one possible state for each sampling period. The transition between each state depends on

Urban	Motorway	Combined	Other
HUTSIM	AUTOBAHN	AIMSUN2	ANATOLL
MICSTRAN	FREEVU	CORSIM	PHAROS
NETSIM	FRESIM	INTEGRATION	SHIVA
PADSIM	MIXIC	PARAMICS	SIMDAC
SITRA-B+	SISTM	TRANSIMS	

Table 1: Types of models

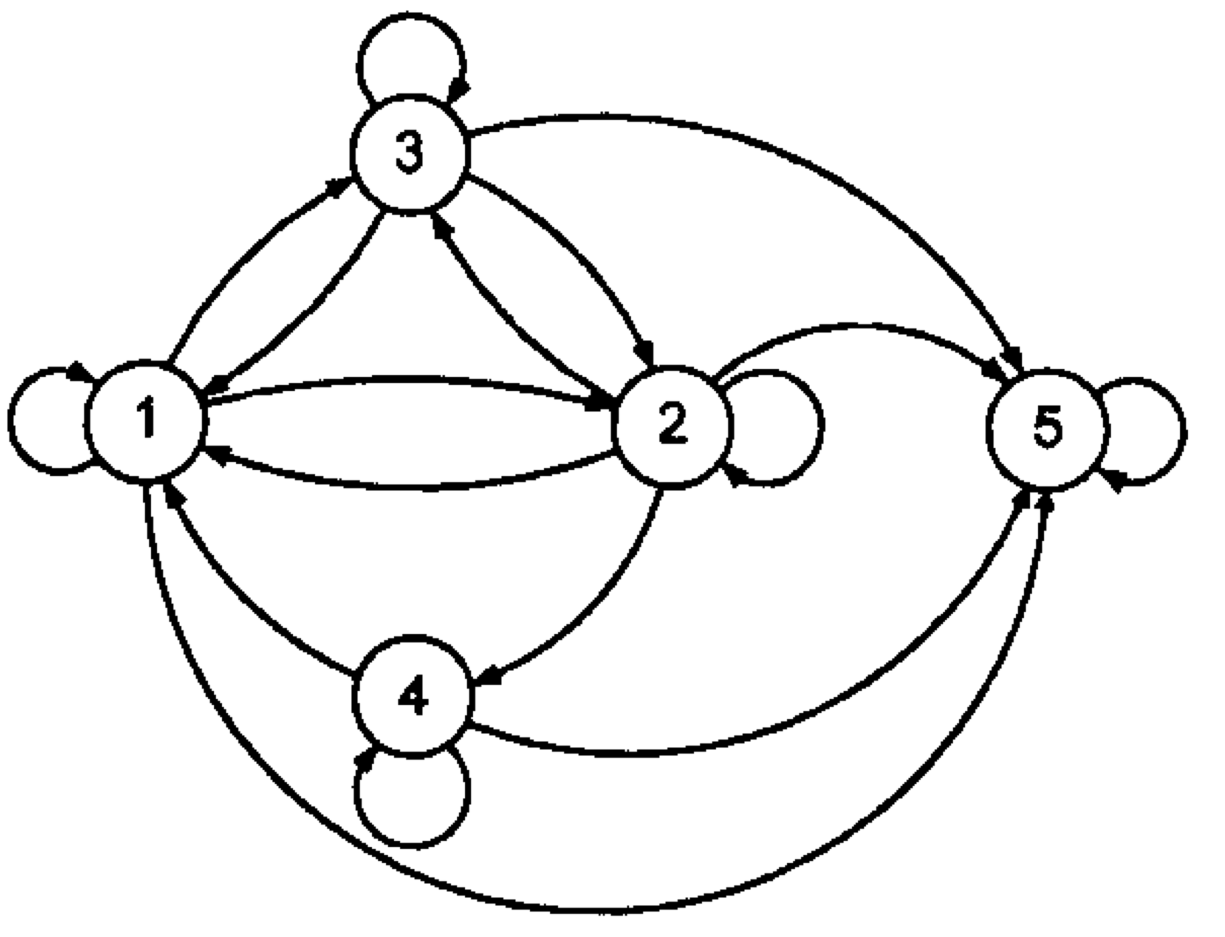


Figure 1: *SITS* state diagram diagram (1-acceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision)

the driver behaviour model and its surrounding environment. Some transitions are not possible; for instance, it is not possible to move from state 4 (stopped) to state 2 (braking), although it is possible to move from state 2 to state 4.

Included on the most important elements of *SITS* are the network components, travel demand, and driving decisions. Network components include the road network geometry, vehicles, and the traffic control. To each driver it is assigned a set of attributes that describe the drivers behaviour, including desired speed, and his profile (e.g., from conservative to aggressive). Likewise, vehicles have their own specifications, including size and acceleration capabilities. Travel demand is simulated using origin destination matrices given as an input to the model.

At this stage of development the *SITS* considers different types of driver behaviour models, namely car following, free flow and lane changing logic. *SITS* considers each vehicle in the network to be in one of two driver regimes: free flow and car-following. The free flow regime prevails when there is either: (*i*) no lead vehicle in front of the subject vehicle or (*ii*) the leading vehicle is sufficiently far ahead that it does not influence the subject vehicles behaviour. In the free flow case the driver travels at his desired maximum speed. Car-following regime dictates acceleration/deceleration decisions when a lead vehicle is near enough to the subject vehicle in order to maintain a safe following distance. Accelerations and decelerations are simulated using the Perception-Driver Model (PDM). In this model, the acceleration of a given driver depends on his speed, on the distance to the leading vehicle and on the speed difference between them.

The lane changing model in *SITS* uses a methodology that tries to mimic a driver behaviour when producing a lane change. This methodology was implemented in three steps: (*i*) decision to consider a lane change; (*ii*) selection of a desired lane; (*iii*) execution of the desired lane change if the gap distances are acceptable. A driver produces a lane change maneuver in order to increase speed, overtake a slower or heavier vehicle or to avoid the lane connected to a ramp. After selecting a lane, the driver examines the lead and lag gaps in the target lane in order to determine if the desired change can be executed. If both the lead and lag gaps are acceptable, the desired lane change is executed in a sampling interval.

The simulation model adopted in the *SITS* is a stochastic one. Some of the processes include random variables such as, individual vehicle speed and input flow. These values are generated randomly according to a pre-defined amplitude interval.

The main types of input data to the simulator are the network description, the drivers and vehicles specifications and the traffic conditions. The output of *SITS* consists not only in a continuously animated graphical representation of the traffic network but also the data gathered by the detectors, originating different types of printouts.

4. Simulation Results and Dynamical Analysis

In the dynamic analysis are applied tools of systems theory. In this line of thought, a set of simulation experiments are developed in order to estimate the influence of the vehicle speed $v(t;x)$, the road length l and the number of lanes n_l in the traffic flow $\phi(t;x)$ at time t and road coordinate x. For a road with n_l lanes the Transfer Function (TF) between the flow measured by two sensors is calculated by the expression:

$$G_{r,k}(s;x_j,x_i) = \Phi_r(s;x_j)/\Phi_k(s;x_i)\,, \tag{1}$$

where k,$r = 1,2, \ldots , n_l$ define the lane number and, x_i and x_j represent the road coordinates ($0 \leq x_i \leq x_j \leq l$), respectively. The Fourier transform for each traffic flow is:

$$\begin{aligned} \Phi_r(s;x_j) &= F\{\phi_r(t;x_j)\}, \\ \Phi_k(s;x_i) &= F\{\phi_k(t;x_i)\}. \end{aligned} \tag{2}$$

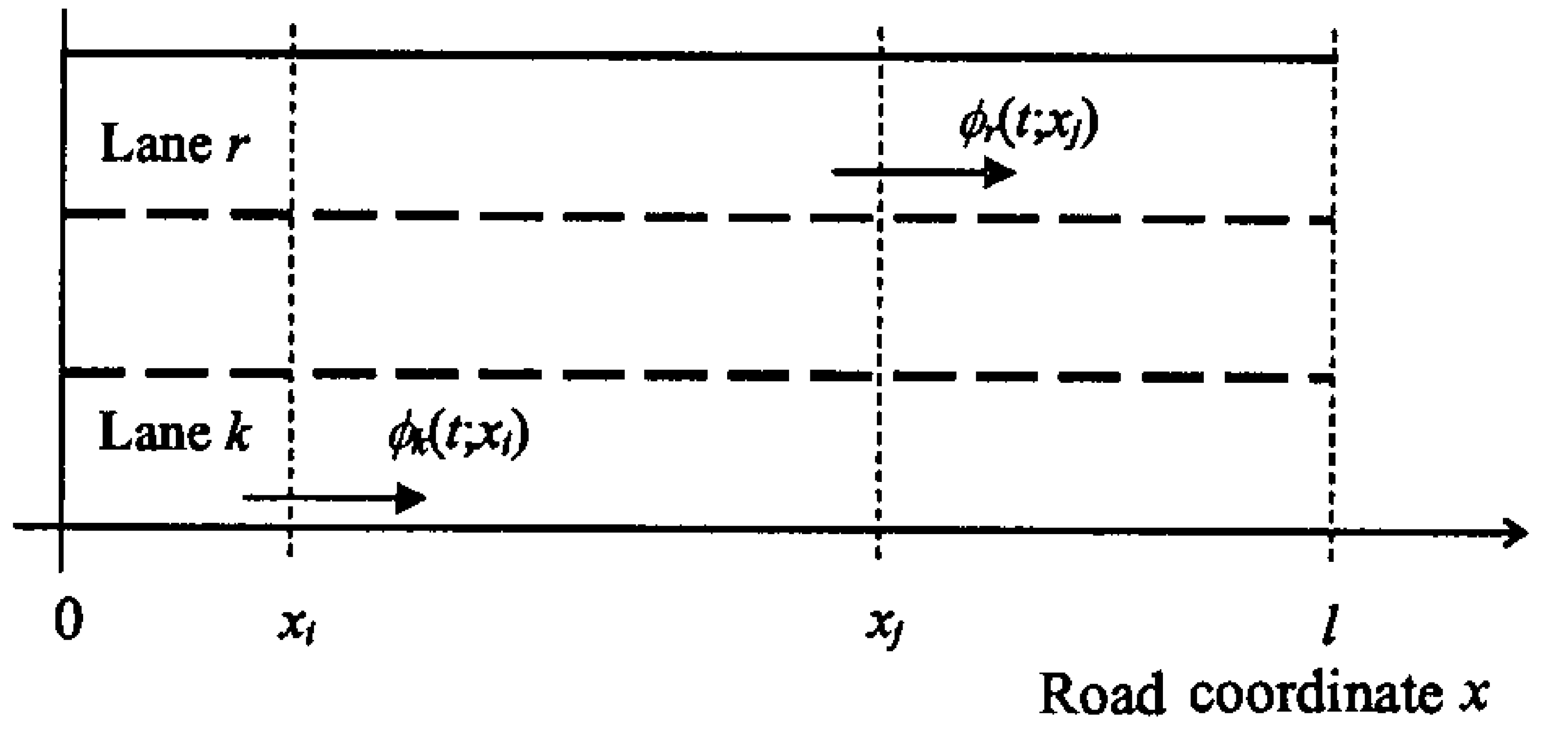


Figure 2: Overall schema of the notation adopted

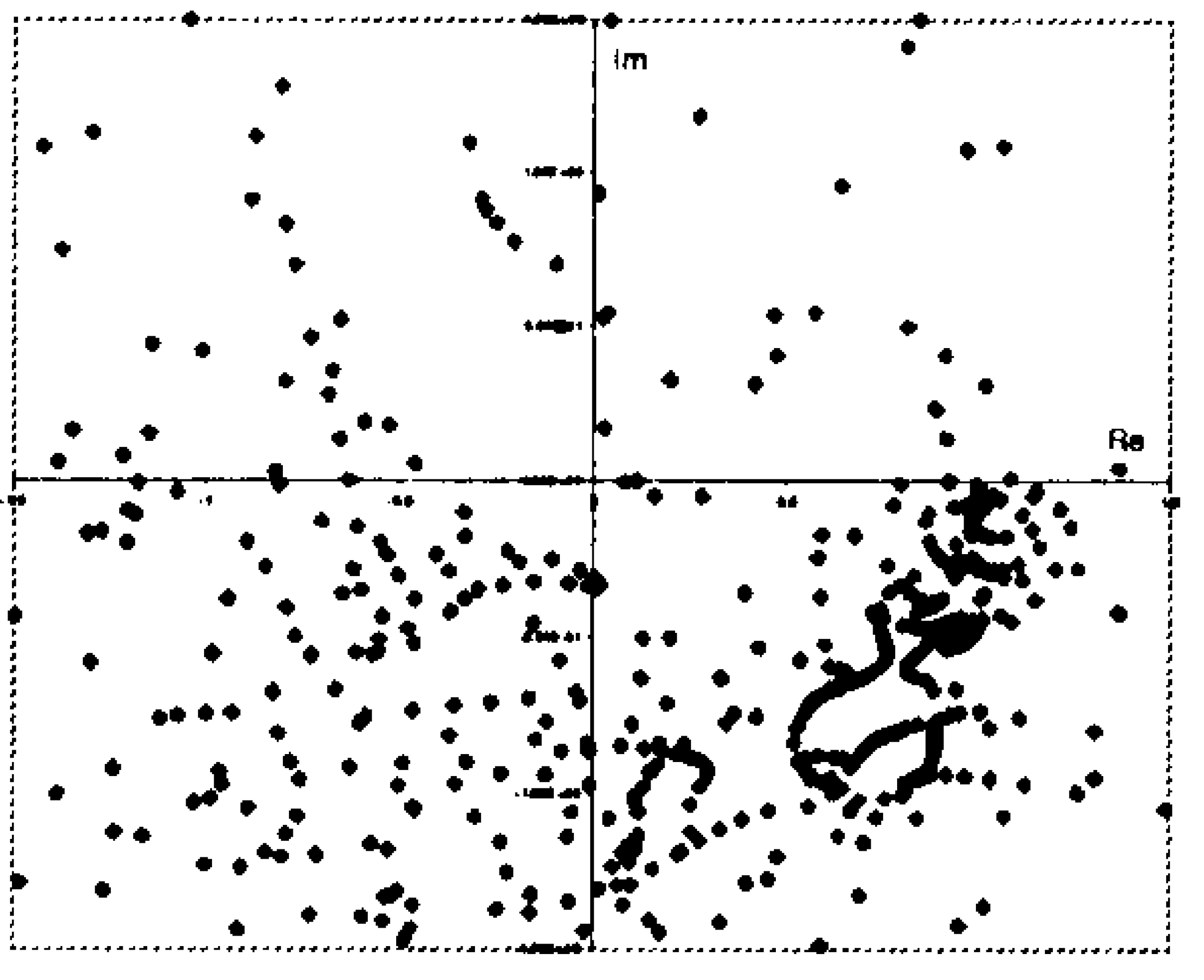
Figure 2 show an overall schema of the notation adopted in the analysis of traffic dynamics.

It should be noted that, traffic flow is a time variant system but, in the sequel, it is shown that the Fourier transform can be used to analyze the system dynamics.

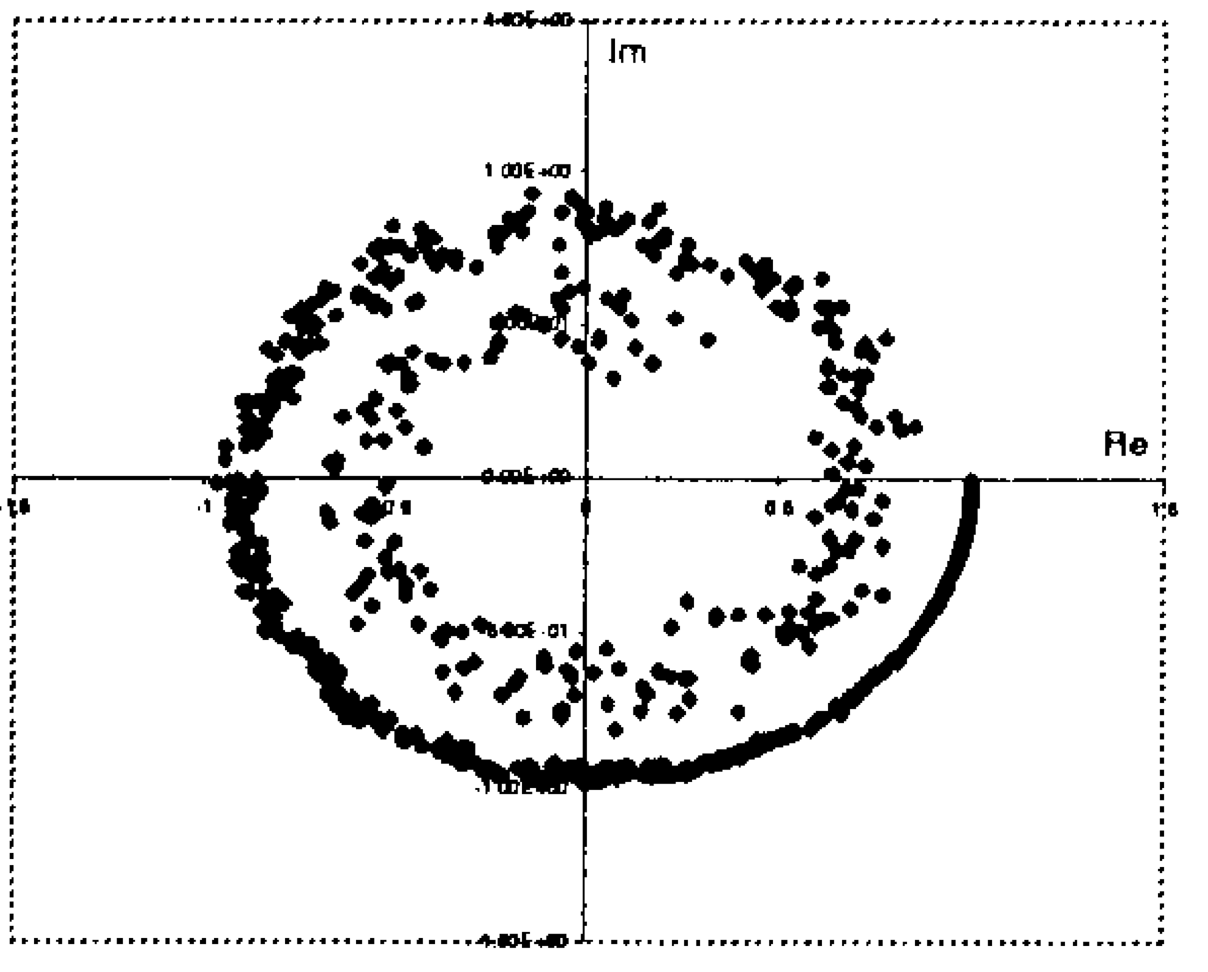
The first group of experiments considers a one-lane road (i.e., $k = r = 1$) with length $l = 1000$ m. Across the road are placed n_s sensors equally spaced. The first sensor is placed at the beginning of the road (i.e., at $x_i = 0$) and the last sensor at the end (i.e., at $x_j = l$). Therefore, we calculate the *TF* between two traffic flows at the beginning and the end of the road such that, $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} for a vehicle speed $v_1(t;0) \in [30, 70]$ km h^{-1}, that is, for $v_1(t;0) \in [v_{av} - \Delta v, v_{av} + \Delta v]$, where $v_{av} = 50$ km h^{-1} is the average vehicle speed and $\Delta v = 20$ km h^{-1} is the maximum speed variation. These values are generated according to a uniform probability distribution function.

Figure 3a shows the polar plot of the *TF* $G_{1,1}(s;1000,0) = \Phi_1(s;1000)/\Phi_1(s;0)$ between the traffic flow at the beginning and end of the one-lane road. It can be observed that the result is distinct from those usual in systems theory revealing a large variability. Moreover, due to the stochastic nature of the phenomena involved different experiments using the same input range parameters result in different *TF*s.

This phenomenon makes the analysis complex and experience demonstrates that efficient tools capable of rendering clear results are still lacking. Moreover, classical models are adapted to "deterministic" tasks, and are not well adapted to the "random" operation that occurs in systems with a non-structured and changing environment.



a)



b)

Figure 3: *a)* Polar plot of *TF* $G_{1,1}(s;1000,0)$ for $n = 1$ experiment and *b)* The *STF* $T_{1,1}(s)$ for $n = 2000$ experiments, with $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} and $v_1(t;0) \in [30, 70]$ km h^{-1}(v_{av} = 50 km h^{-1} Δv= 20 km h^{-1}, l = 1000 m and $n_l = 1$)

In order to overcome the problems, alternative concepts are required. Statistics is a mathematical tool well adapted to handle a large volume of data but not capable of dealing with time-dependent relations. Therefore, to overcome the limitations of statistics, it is adapted a new method [8], that takes advantage of the Fourier transform by embedding both tools.

In this line of thought, the first stage of the new modelling formalism starts by comprising a set of input variables that are free to change independently (*ivs*) and a set of output variables that depend on the previous ones (*ovs*). In a traffic system the *ivs* and *ovs* are defined as $\phi_k(t; x_i)$ and $\phi_r(t; x_j)$, that is the traffic flows at positions x_i and x_j, respectively, at time t and for the k, r-th lanes ($k, r = 1, 2, 3, \dots, n_l$).

The second stage of the formalism consists on embedding the statistical analysis into the Fourier transform through the algorithm:

i) A statistical sample is obtained by carrying out a large number (n) of experiments having appropriate time/space evolutions. All the *ivs* and *ovs* are calculated and sampled in the time domain.

ii) The Fourier transform is computed for each of the *ivs* and *ovs*.

iii) Statistical indices are calculated for the Fourier spectra obtained in *ii)*.

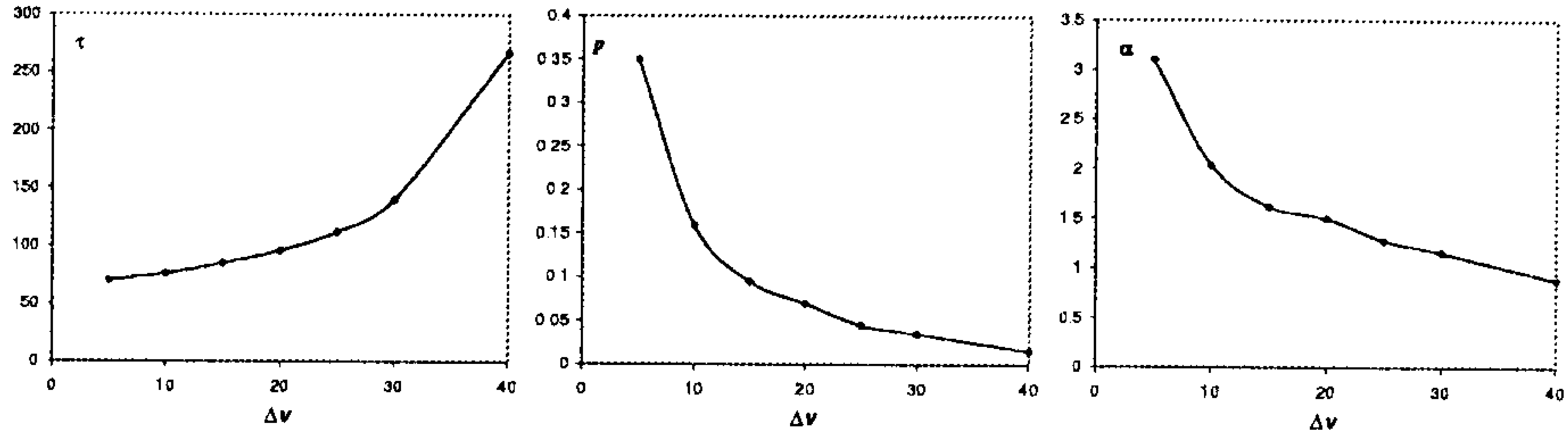


Figure 4: Time delay τ, pole p and fractional order α versus Δv for $v_{av} = 50$ km h^{-1}, $n_l = 1$, $l = 1000$ m, $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}

iv) The values of the statistical indices calculated in *iii)* (for all the variables and for each frequency) are collected on a "composite" frequency response entitled Statistical Transfer Function (*STF*) of each *TF*.

The previous procedure may be repeated for different numerical parameters (e.g., traffic flow, vehicle speed, road geometry) and the partial conclusions integrated in a broader paradigm [2].

To illustrate the proposed modelling concept, was repeated the previous simulation for a sample of $n = 2000$ and it was observed the existence of a convergence of the *STF*, $T_{1,1}(s; 1000, 0)$, as show in Figure 3*b*, for a one-lane road with length $l = 1000$ m, $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1} and $v_1(t;0) \in [30, 70]$ km h^{-1}.

Based on this result we can approximate numerically the *STF* to a fractional order system [3] with time delay yielding the approximate expression of the type:

$$T_{1,1}(s; 1000, 0) = \frac{k_B \; e^{-\tau s}}{\left(\frac{s}{p} + 1\right)^{\alpha}} . \tag{3}$$

For the numerical parameters of Figure 3b we get $k_B = 1.0$, $\tau = 96.0$ sec, $p = 0.07$ and $\alpha = 1.5$.

The parameters (τ, p, α) vary with the average speed v_{av} and its range of variation Δv, the road length l and the input vehicle flow $\phi_k(t;0)$. Figure 4 shows (τ, p, α) versus Δv for $v_{av} = 50$ km h^{-1}.

It is interesting to note that $(\tau, p) \to (\infty,\ 0)$, when $\Delta v \to v_{av}$, and $(\tau, p) \to (l\ {v_{av}}^{-1},\ \infty)$ when $\Delta v \to 0$. These results are consistent with

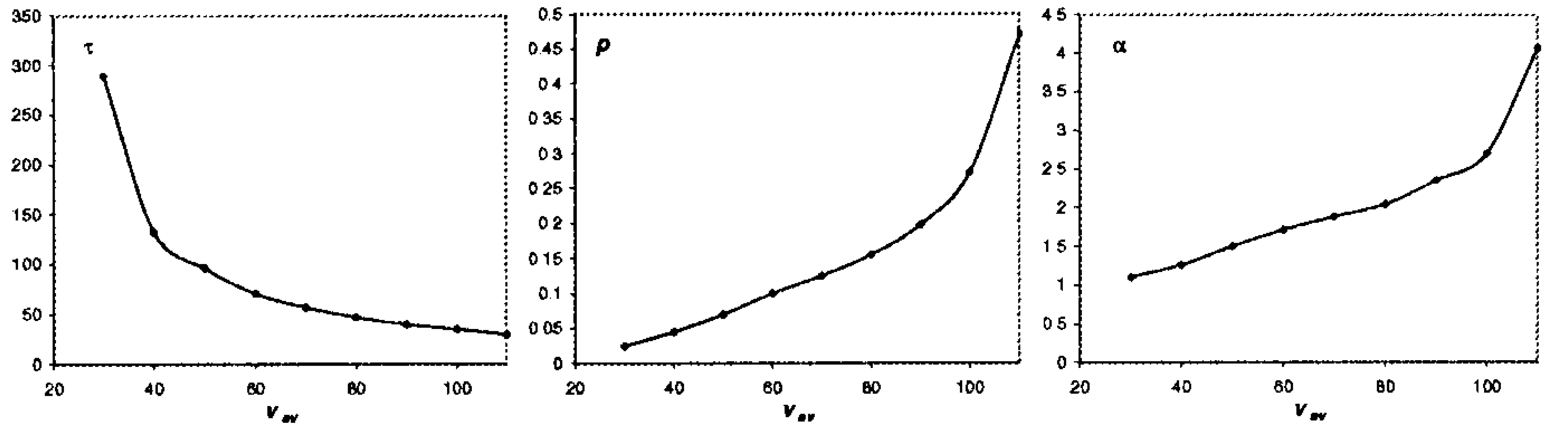


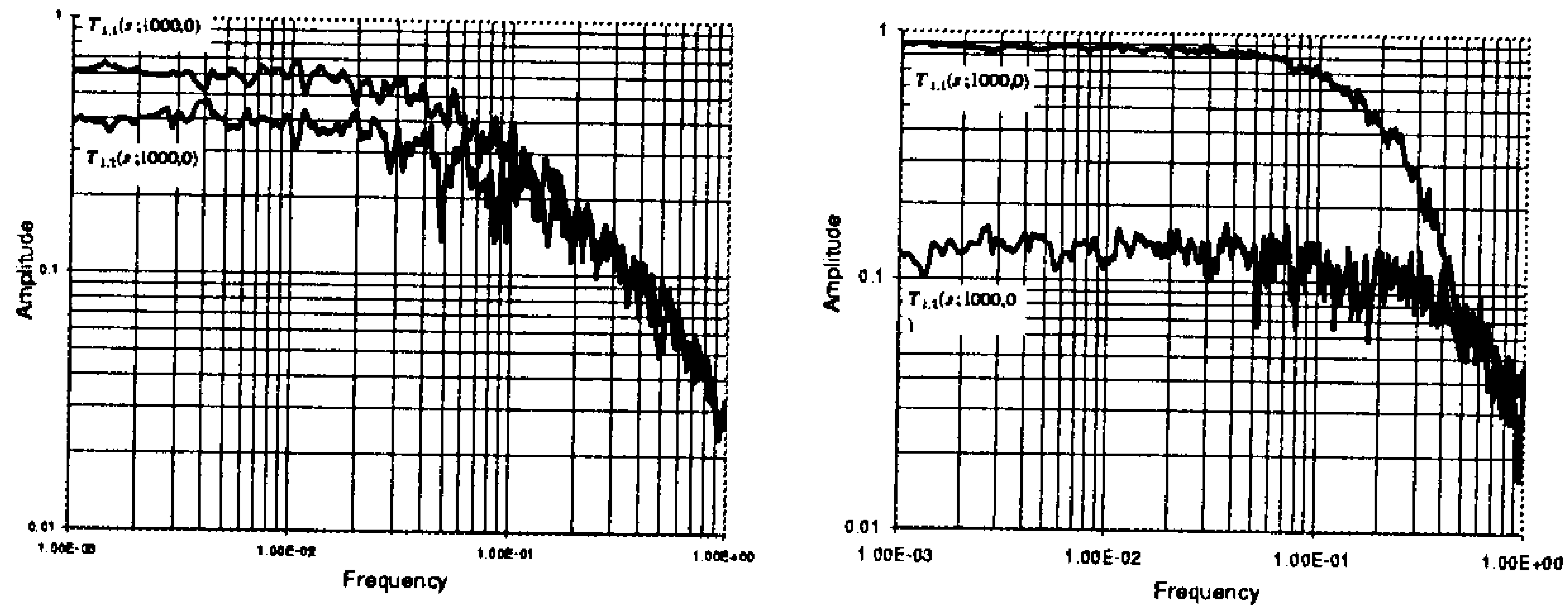
Figure 5: Time delay τ, pole p and fractional order α versus v_{av} for $\Delta v = 20$ km h^{-1}, $n_l = 1$, $l = 1000$ m, $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}.

our experience that suggests a pure transport delay $T(s) \approx e^{-\tau s}$ ($\tau = l\ {v_{av}}^{-1}$), when $\Delta v \to 0$, and $T(s) \approx 0$, when $\Delta v \to v_{av}$ (because of the existence of a blocking cars, with zero speed, on the road). On the other hand, Figure 5 shows (τ, p, α), versus v_{av} for $\Delta v = 20$ km h^{-1}. In this case we have $(\tau, p) \to (\infty,\ 0)$, when $v_{av} \to \Delta v$, and $(\tau, p) \to (\ 0, \infty)$ when $v_{av} \to \infty$, which has a similar intuitive interpretation.

In a second group of experiments are analyzed the characteristics of the *STF* matrix for roads with two lanes considering identical traffic conditions (i.e., $\phi_k(t;0) \in [1, 8]$ vehicles s^{-1}, k = 1,2, l = 1000, Δv= 20 km h^{-1}).

Figure 6a and 6b depict the amplitude Bode diagram of $T_{1,1}(s; 1000, 0)$ and $T_{1,2}(s; 1000, 0)$ for $v_{av} = 50$ km h^{-1} (i.e., $v_k(t;0) \in [30, 70]$ km h^{-1}), and for $v_{av} = 90$ km h^{-1} (i.e., $v_k(t;0) \in [70, 110]$ km h^{-1}), respectively.We verify that $T_{1,1}(s;1000,0) \approx T_{2,2}(s;1000,0)$ and $T_{1,2}(s;1000,0) \approx T_{2,1}(s;1000,0)$. This property occurs because *SITS* uses a lane change logic where, after the overtaking, the vehicle tries to return to the previous lane. Therefore, lanes 1 and 2 have the same characteristics leading to identical *STF*.

Comparing Figure 6a and 6b, we conclude that the transfer matrix elements vary significantly with v_{av}. Moreover, the *STF* parameter dependence is similar to the one-lane case represented previously. Figure 7 and Figure 8 show the variation of the parameters (k_B, p, α) for $T_{1,1}(s;1000,0)$ versus Δv, with $v_{av} = 50$ km h^{-1}, and versus v_{av}, with $\Delta v = 20$ km h^{-1}, for $n_l = 2$, respectively.



a) *b)*

Figure 6: Amplitude Bode diagram for $T_{r,k}$(s;1000,0), n_l = 2, l = 1000 m, ϕ_k(t;0) $\in$ [1, 8] vehicles s^{-1}, $\Delta v = 20$ km h^{-1}, k = 1,2: *a)* $v_{av} = 50$ km h^{-1} *b)* $v_{av} = 90$ km h^{-1}

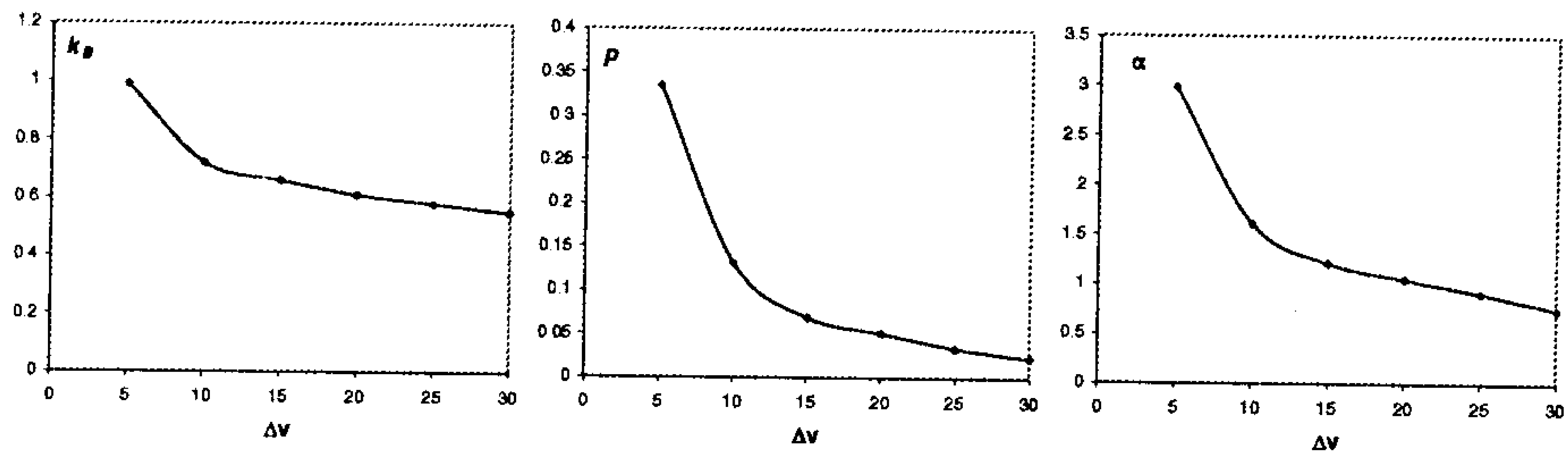


Figure 7: Gain k_B, pole p and fractional order α versus Δv for $v_{av} = 50$ km h^{-1}, $n_l = 2$, $l = 1000$ m and $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}

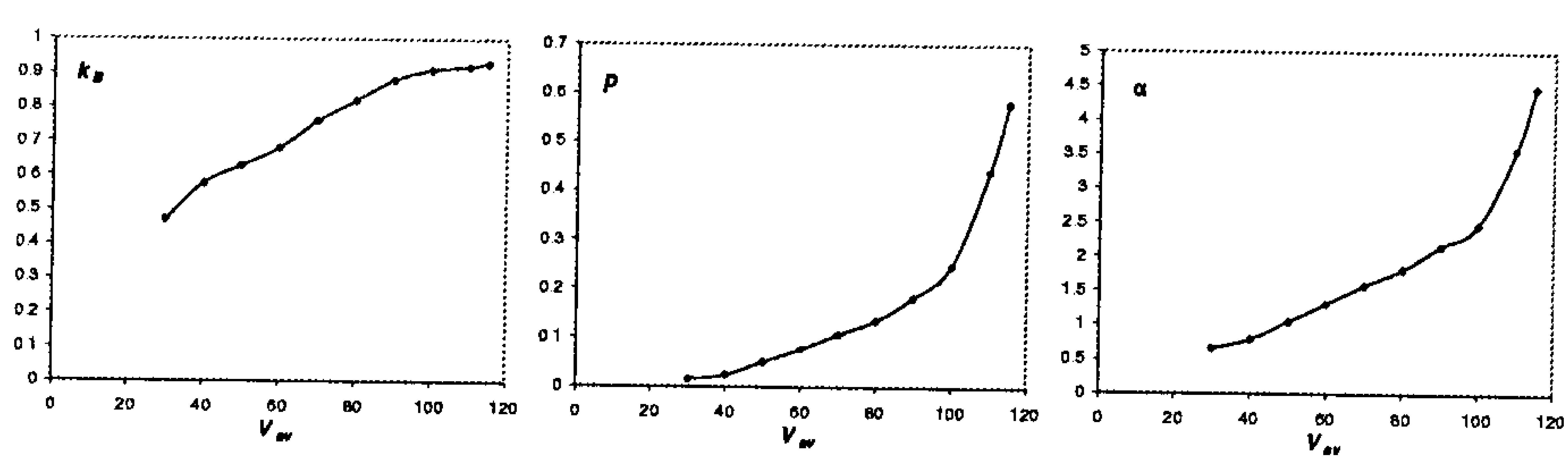


Figure 8: Gain k_B, pole p and fractional order α versus v_{av} for $\Delta v = 20$ km h^{-1}, $n_l = 2$, $l = 1000$ m and $\phi_1(t;0) \in [1, 8]$ vehicles s^{-1}.

We conclude that:

i) The time delay τ is independent of the number of lanes n_l.

ii) For a fixed set of parameters, in each *STF*, we have gain × bandwidth ≈ constant

iii) For each row of the transfer matrix, the sum of the *STF* gains is the unit.

iv) The gains and the poles of the diagonal elements of the *STF* matrix are similar. The gain of the non-diagonal elements, that represent dynamic coupling between the lanes, are lower (due to *iii*), but the corresponding pole are higher (due to *ii*).

v) The fractional order α increases with v_{av}. Nevertheless, the higher the number of lanes the lower the low-pass filter effect, that is, the smaller the value of α.

5. Conclusions

One of the most important proceedings of mathematical simulation and modelling is the translation of real contexts and objects into parameters that can be represented numerically. This is the point at which many of the inaccuracies and errors are introduced. Therefore, further research on ITS dynamic models and parameter identification is still needed.

In this paper was described a software tool based on a microscopic simulation approach, to reproduce real traffic conditions in an urban or non-urban network. At this stage of development the SITS considers different types of driver behaviour model, namely car following, free flow and lane changing logic. On the next stage of development we will include better driver behaviour models and traffic safety models. Another important improvement is the inclusion of aspects such as, ramp-metering and signal control devices.

Several experiments were carried out in order to analyze the dynamics of the traffic systems. The results of using classical system theory tools point out that it is possible to develop traffic systems, including the knowledge gathered with automatic control algorithms. In his line of thought it was also presented a new modelling formalism based on the embedding of statistics and Fourier transform.

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