Analysis of Robot Dynamics and Compensation Using Classical and Computed Torque Techniques

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Abstract— **A classical analysis of the dynamics of robot manipulators is presented. It is shown that these systems have configuration-dependent properties and can be open-loop unstable. Due to this fact, present day linear controllers are inefficient. On the other hand, nonlinear hardware and software compensation methods also are shown to have some limitations. Controllers based on the direct design algorithm and the computed torque method have superior performances. These algorithms have nonlinear loops yet, our paper shows that a linear analysis is still feasible. Therefore, classical design tools can be adopted in order to develop practical implementations.**

VI. Introduction

THE dynamics of robot manipulators is highly nonlinear which makes difficult their efficient control. Classical control methods are well known; however, they are inadequate in the presence of strong nonlinearities. On the other hand, nonlinear controllers [1]–[4] produce better results but the nonlinear analysis and design is not as systematic and clear as the linear case. Some work has been done on relating linear methods to manipulator dynamics [5]–[14]. However, the complexity of the problem has not allowed yet methods which permit general conclusions to be drawn about stability, imperfect modeling effects, etc. This paper intends to link classical linear methods with robot modern nonlinear control schemes. Having this idea in mind we organize the paper as follows. In Section II we analyze the dynamics of a two degrees of freedom (d.o.f.) manipulator from a classical (Laplace-based) point of view. Using this approach we derive a set of transfer functions (TF's) that characterize the dynamics of robot manipulators. The TF's reveal that manipulating systems are intrinsically unstable. Therefore, in order to render the system stable, we need appropriate compensation techniques. In this line of thought, in Section III, we analyze both hardware and software compensation methods. These compensations have limitations which make necessary the development of complementary control strategies. In Section IV we analyze, from a classical perspective, model-based nonlinear algorithms that accomplish not only a dynamic compensation but also the control action. Finally, in Section V, conclusions are drawn.

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VII. Dynamics of a Two Degrees of Freedom Manipulator

The dynamic equations of the two d.o.f. manipulator (Fig. 1) can be easily obtained from the Lagrangian [15], [16]:

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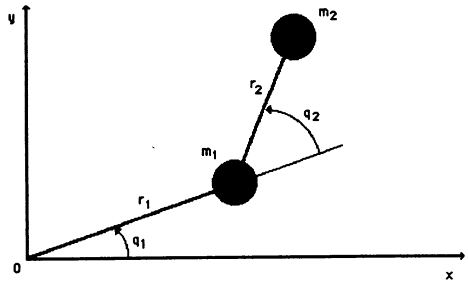


Fig. 1. The two link manipulator.

$$\mathbf{T}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right)-\frac{\partial L}{\partial \mathbf{q}} \quad (1a)$$

$$L=K-P. \quad (1b)$$

For this system we have [17], [18]:

$$\mathbf{T}=\mathbf{J}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{G}(\mathbf{q}) \quad (2)$$

where

$$\mathbf{J}(\mathbf{q})=\begin{bmatrix} m_1r_1^2+m_2(r_1^2+r_2^2+2r_1r_2C_2)+J_1 & m_2(r_2^2+r_1r_2C_2) \\ m_2(r_2^2+r_1r_2C_2) & m_2r_2^2+J_2 \end{bmatrix} \quad (3a)$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})=\begin{bmatrix} -2m_2r_1r_2S_2\dot{q}_1\dot{q}_2-m_2r_1r_2S_2\dot{q}_2^2 \\ m_2r_1r_2S_2\dot{q}_1^2 \end{bmatrix} \quad (3b)$$

$$\mathbf{G}(\mathbf{q})=\begin{bmatrix} g[m_1r_1C_1+m_2(r_2C_{12}+r_1C_1)] \\ gm_2r_2C_{12} \end{bmatrix}. \quad (3c)$$

Considering small variations in the neighborhood of an equilibrium point, that is:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}=\begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix}+\begin{bmatrix} \delta T_1 \\ \delta T_2 \end{bmatrix} \quad (4a)$$

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}=\begin{bmatrix} q_{10} \\ q_{20} \end{bmatrix}+\begin{bmatrix} \delta q_1 \\ \delta q_2 \end{bmatrix} \quad (4b)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}+\begin{bmatrix} \delta\dot{q}_1 \\ \delta\dot{q}_2 \end{bmatrix} \quad (4c)$$

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}+\begin{bmatrix} \ddot{q}_1 \\ \delta\ddot{q}_2 \end{bmatrix} \quad (4d)$$

and substituting in (3), then it comes:

$$\begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix}=g\begin{bmatrix} m_1r_1C_{10}+m_2(r_1C_{10}+r_2C_{120}) \\ m_2r_2C_{120} \end{bmatrix} \quad (5a)$$

$$\begin{bmatrix} \delta T_1 \\ \delta T_2 \end{bmatrix}=\begin{bmatrix} m_1r_1^2+m_2(r_1^2+r_2^2+2r_1r_2C_{20})+J_1 & m_2(r_2^2+r_1r_2C_{20}) \\ m_2(r_2^2+r_1r_2C_{20}) & m_2r_2^2+J_2 \end{bmatrix} \cdot \begin{bmatrix} \delta\ddot{q}_1 \\ \delta\ddot{q}_2 \end{bmatrix} - g\begin{bmatrix} m_1r_1S_{10}+m_2(r_1S_{10}+r_2S_{120}) & m_2r_2S_{120} \\ m_2r_2S_{120} & m_2r_2S_{120} \end{bmatrix} \cdot \begin{bmatrix} \delta q_1 \\ \delta q_2 \end{bmatrix}. \quad (5b)$$

n	Number of degrees of freedom
L	Lagrangian
K	Kinetic energy
P	Potential energy
g	Acceleration at the gravity field
$\mathbf{q}$	n vector of joint positions
$\dot{\mathbf{q}}$	n vector of joint velocities
$\ddot{\mathbf{q}}$	n vector of joint accelerations
$\mathbf{q_d}$	n vector of desired joint positions
$\dot{\mathbf{q}}_\mathbf{d}$	n vector of desired joint velocities
$\ddot{\mathbf{q}}_\mathbf{d}$	n vector of desired joint accelerations
$\mathbf{q_0}$	n vector of joint bias positions
$\dot{\mathbf{q}}_\mathbf{0}$	n vector of joint bias velocities
$\ddot{\mathbf{q}}_\mathbf{0}$	n vector of joint bias accelerations
$\delta\mathbf{q}$	n vector of small amplitude joint position deviations
$\delta\dot{\mathbf{q}}$	n vector of small amplitude joint velocity deviations
$\delta\ddot{\mathbf{q}}$	n vector of small amplitude joint acceleration deviations
$\mathbf{T}$	Joint n vector torque of the uncompensated system
$\mathbf{T}^*$	Joint n vector torque of the compensated system
$\mathbf{T_C}$	Joint n vector compensation torque
$\mathbf{J}(\mathbf{q})$	Positive-definite, symmetric inertial $n \times n$ matrix of the uncompensated system
$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$	Coriolis/centripetal n vector torque of the uncompensated system
$\mathbf{G}(\mathbf{q})$	Gravitational n vector torque of the uncompensated system
$\mathbf{J_C}(\mathbf{q})$	$n \times n$ inertial matrix of the compensation structure
$\mathbf{C_C}(\mathbf{q}, \dot{\mathbf{q}})$	Coriolis/centripetal n vector torque of the compensation structure
$\mathbf{G_C}(\mathbf{q})$	Gravitational n vector torque of the compensation structure
$\delta\mathbf{T}$	n vector of the error torque due to imperfect compensation
l_i	Length of link i
r_i	Distance from axis i of the center of gravity of link i
m_i	Mass of link i
J_i	Inertia of the actuator of link i
R_i	Distance from axis i of the center of gravity of the counterweight of link i
M_i	Mass of the counterweight of link i
s	Laplace variable
$\mathcal{L}$	Laplace transform operator
$\mathbf{G}(s)$	$n \times n$ transfer matrix of the linearized robot inverse dynamics
$d(s)$	Characteristic equation of the linearized open loop system
$C_i(s)$	PID controller for joint i
$H_i(s)$	Feedback gain of joint i
$d'(s)$	Characteristic equation of the linearized closed loop system
$\mathbf{K_1}, \mathbf{K_2}$	Diagonal feedback $n \times n$ matrices of constant, for the velocity and position feedback gains, respectively
$\mathbf{D}$	Diagonal $n \times n$ matrix of constant for the position direct gain
$P(s)$	Perturbation signal corresponding to the error torque $\delta\mathbf{T}$

A system description in the s-plane applying the Laplace

$$C_{10} = \cos(q_{10}), C_{20} = \cos(q_{20}),$$
$$C_{120} = \cos(q_{10} + q_{20}), C_1 = \cos(q_1),$$
$$C_2 = \cos(q_2), C_{12} = \cos(q_1 + q_2)$$
$$S_{10} = \sin(q_{10}), S_{20} = \sin(q_{20}),$$
$$S_{120} = \sin(q_{10} + q_{20}), S_1 = \sin(q_1),$$
$$S_2 = \sin(q_2), S_{12} = \sin(q_1 + q_2).$$

transform to (5) leads to:

$$\mathbf{T}(\mathbf{s}) = \mathbf{G}(\mathbf{s})\,\mathbf{Q}(\mathbf{s}) \tag{6a}$$
$$G_{11}(s) = [m_1 r_1^2 + m_2(r_1^2 + r_2^2 + 2r_1 r_2 C_{20}) + J_1]s^2 - g[m_1 r_1 S_{10} + m_2(r_1 S_{10} + r_2 S_{120})] \tag{6b}$$
$$G_{12}(s) = G_{21}(s) = [m_2(r_2^2 + r_1 r_2 C_{20})]s^2 - g m_2 r_2 S_{120} \tag{6c}$$
$$G_{22}(s) = (m_2 r_2^2 + J_2)s^2 - g m_2 r_2 S_{120} \tag{6d}$$

where $\mathbf{T}(s) = \mathcal{L}\{\delta\mathbf{T}\}$ and $\mathbf{Q}(s) = \mathcal{L}\{\delta\mathbf{q}\}$. Equation (6) constitutes the so-called inverse system description. The direct description yields:

$$\mathbf{Q}(\mathbf{s}) = \mathbf{G}(\mathbf{s})^{-1}\mathbf{T}(\mathbf{s}) = \frac{\mathbf{N}(\mathbf{s})}{d(s)}\mathbf{T}(\mathbf{s}) \tag{7a}$$
$$N_{11}(s) = G_{22}(s) \tag{7b}$$
$$N_{12}(s) = N_{21}(s) = -G_{12}(s) \tag{7c}$$
$$N_{22}(s) = G_{11}(s) \tag{7d}$$
$$d(s) = N_{11}(s)\,N_{22}(s) - [N_{12}(s)]^2. \tag{7e}$$

Analyzing these equations we conclude that [19], [20]:

- In stability terms, Coriolis/centripetal terms are of second order influence on the dynamic response. In other words, only the inertial and gravitational terms influence the TF's poles and zeros.
- Due to the nonzero value of N_{ij} $(i \neq j)$ there is coupling between the outputs.
- Pole and zero variations are a result of the inertial and gravitational dependence on q_{10} and q_{20}. With a manipulator in outer space, where gravity loading is absent, or with manipulators having a horizontal structure, such as the SCARA robots, the system has two poles fixed at the origin of the s-plane, and variable gain due to the variation of the inertial terms.
- The polynomials $d(s)$ and $N_{ij}(s)\,(i,j = 1,2)$ have no odd powers of s. Therefore, their roots occur in pairs, either on the imaginary axis or on the real axis, with equal magnitudes and opposite signs.
- To each downward link corresponds a complex conjugate pole pair, lying on the imaginary axis. To each upward link, corresponds a real pole pair, symmetrical about the origin (Fig. 2). Therefore, robot manipulators with downward links have *a priori* better performance [21]–[23].

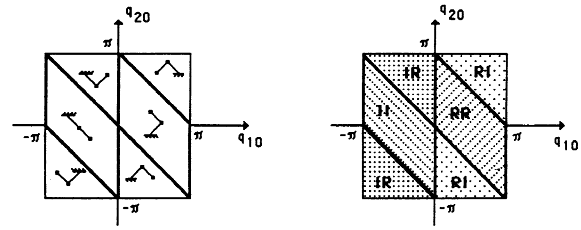


Fig. 2. Locus of the roots of the characteristic equation $d(s)$ for the two link manipulator. I = pair of symmetrical pure imaginary roots, R = pair of symmetrical real root, and — = pair of zero roots.

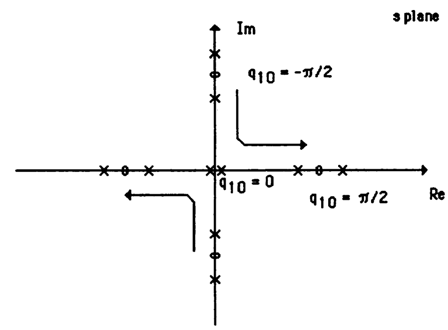


Fig. 3. Loci of the poles and zeros of $G_{11}(s)$ in the s-plane, with $q_{10} \in [-\pi/2, \pi/2]$ and $q_{20} = 0$.

- The magnitude of the poles increases when each link approaches the vertical and decreases towards zero when approaching the horizontal.
- Manipulators with n d.o.f. have $2n$ poles and $2(n-1)$ zeros, except when k links are horizontal. In this case, the system becomes lower order with $2(n-k)$ poles and $2(n-k-1)$ zeros when $n > k$ or only 2 poles when $n = k$.

Fig. 3 represents the locus of the poles and zeros of $G_{11}(s)$ when both links are aligned. The picture for the other TF's are similar. The existence of (linear) mechanical damping, not considered in our modeling, produces a slight improvement on the system stability by pushing the poles towards the left half s-plane.

Concerning the control of the two link manipulator several conclusions can now be drawn. A natural approach is to implement a PID control scheme including a position/velocity feedback on each joint (Fig. 4) which is, in fact, the standard industrial control scheme. Based on this controller structure and on the proposed model (7) we get the expressions:

$$\begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} = \frac{1}{d'(s)} \begin{bmatrix} C_1(N_{11} + C_2 H_2) & C_2 N_{21} \\ C_1 N_{12} & C_2(N_{22} + C_1 H_1) \end{bmatrix} \cdot \begin{bmatrix} Q_{1d}(s) \\ Q_{2d}(s) \end{bmatrix} \tag{8a}$$

$$d'(s) = d(s) + C_1 N_{11} H_1 + C_2 N_{22} H_2 + C_1 C_2 H_1 H_2 \tag{8b}$$

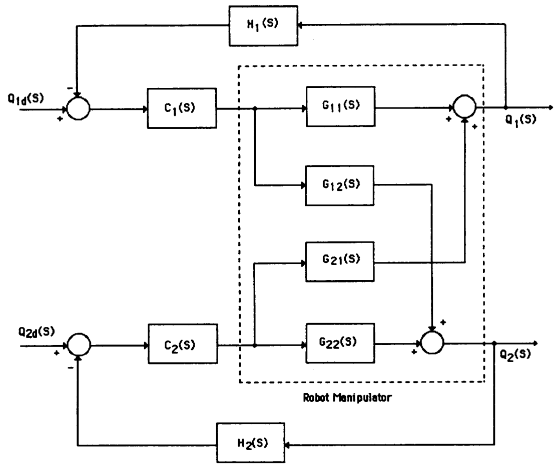


Fig. 4. Present day industrial robot control system.

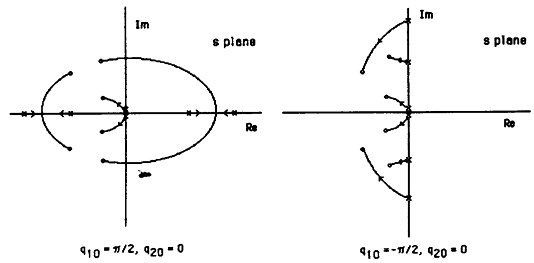


Fig. 5. Possible loci in the s -plane of the closed loop poles for the controller presented in Fig. 4.

Fig. 5 shows a set of loci for a particular choice of parameters of TF's C_1, C_2, H_1 and H_2. We verify a certain degree of overlapping for the closed-loop dominant poles, showing that feedback desensitizes, to some extent, the system to variations in q_{10} and q_{20}. Therefore, this control action makes the system outputs have some resemblances with linear ones [24]. However, for an n d.o.f. manipulator, the designer is faced with the selection of $5n$ interacting parameters without having any systematic criteria for their selection! As n increases the problem soon becomes intractable. Furthermore, a set of satisfactory parameter values for a given region may prove totally unsatisfactory elsewhere, as it sometimes occurs with present day industrial robots whose behavior is "shaky" in some operating regions. Therefore, more efficient control structures must be capable of achieving a systematic and easier adjustment of the controller parameter values. Better performances can be attained using both hardware and software compensation techniques. These strategies will be the topic of the next section.

VIII. Compensation of the Manipulator Dynamics

Because of the poles and zeros of the TF's change with the robot configuration a compensation is required that adjusts itself to the variations of the system dynamics. Such compensation can be implemented at the hardware (i.e., mechanical) level or, alternatively, at the software (i.e., computer) level. We begin by studying the hardware compensation. Then, based on this intuitive approach, we present some concepts towards an alternative software compensation.

A. Hardware Compensation

Several mechanical structures have been used to provide dynamic compensation [25]–[31]. In this paper we shall study the use of counterweights because of its simplicity. Using the Lagrangian for this structure (Fig. 6) we have: [see (9a) below]

$$\mathbf{C}^*(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} -2(m_2r_2 - M_2R_2)r_1S_2\dot{q}_1\dot{q}_2 \\ \quad -(m_2r_2 - M_2R_2)r_1S_2\dot{q}_2^2 \\ (m_2r_2 - M_2R_2)r_1S_2\dot{q}_1^2 \end{bmatrix} \tag{9b}$$

$$\mathbf{G}^*(\mathbf{q}) = \begin{bmatrix} g[(m_1r_1 - M_1R_1)C_1 + (m_2r_2 - M_2R_2)C_{12} \\ \quad +(m_2 + M_2)r_1C_1] \\ g(m_2r_2 - M_2R_2)C_{12} \end{bmatrix} \tag{9c}$$

If:

$$m_1r_1 = M_1R_1 \tag{10a}$$

$$m_2r_2 = M_2R_2 \tag{10b}$$

we can simplify (9), getting: [see (11a) below]

$$\mathbf{C}^*(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{11b}$$

$$\mathbf{G}^*(\mathbf{q}) = \begin{bmatrix} gr_1(m_2 + M_2)C_1 \\ 0 \end{bmatrix}. \tag{11c}$$

$$\mathbf{J}^*(\mathbf{q}) = \begin{bmatrix} m_1r_1^2 + m_2(r_1^2 + r_2^2) & m_2r_2^2 + M_2R_2^2 \\ \quad +M_1R_1^2 + M_2(r_1^2 + R_2^2) & \quad +(m_2r_2 - M_2R_2)r_1C_2 \\ \quad +2(m_2r_2 - M_2R_2)r_1C_2 + J_1 & \\ m_2r_2^2 + M_2R_2^2 & m_2r_2^2 + M_2R_2^2 + J_2 \\ \quad +(m_2r_2 - M_2R_2)r_1C_2 & \end{bmatrix} \tag{9a}$$

$$\mathbf{J}^*(\mathbf{q}) = \begin{bmatrix} m_1r_1^2 + m_2(r_1^2 + r_2^2) & m_2r_2^2 + M_2R_2^2 \\ \quad +M_1R_1^2 + M_2(r_1^2 + R_2^2) + J_1 & \\ m_2r_2^2 + M_2R_2^2 & m_2r_2^2 + M_2R_2^2 + J_2 \end{bmatrix} \tag{11a}$$

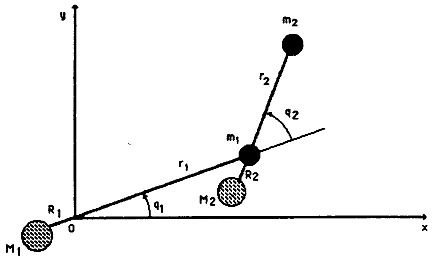


Fig. 6. The two link manipulator with a counterweight structure.

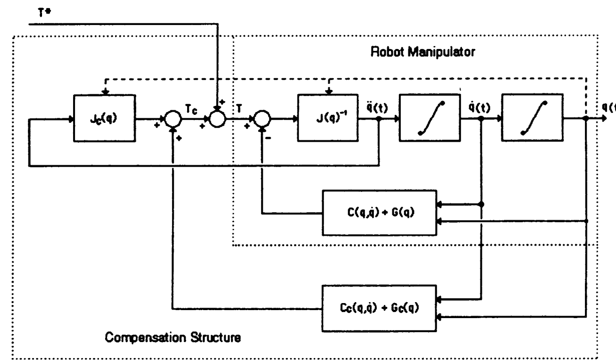


Fig. 7. Block diagram corresponding to the counterweight structure.

Comparing (9) and (11) we verify that [32]:

- The gravitational terms are partially compensated.
- The inertial position dependent terms cancel but new constant terms appear.
- The Coriolis/centripetal terms cancel.

Repeating our linear analysis we observe an improvement in the location of the poles and zeros of the new TF:

$$\begin{bmatrix} T_1^*(s) \\ T_2^*(s) \end{bmatrix} = \begin{bmatrix} [m_1 r_1^2 + m_2(r_1^2 + r_2^2) + M_1 R_1^2 + M_2(r_1^2 + R_2^2) + J_1]s^2 - g[r_1(m_2 + M_2)C_{10}] & (m_2 r_2^2 + M_2 R_2^2)s^2 \\ (m_2 r_2^2 + M_2 R_2^2)s^2 & (m_2 r_2^2 + M_2 R_2^2 + J_2)s^2 \end{bmatrix} \begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix}. \quad (12)$$

From this expression we observe that $Q_2(s)/T_2^*(s)$ may be still unstable due to the incomplete cancellation of the gravitational terms in (11). Furthermore, the new higher inertial terms in (11a) decrease the manipulator bandwidth and therefore, they reduce the manipulator speed.

In conclusion, in order to have a more efficient compensation we must look for a method that:

- Eliminates, completely, the gravitational terms
- Cancels the position dependent inertial terms and, if possible, decreases the constant ones.

B. Software Compensation

We can formally describe the hardware compensation as:

$$\mathbf{T} = \mathbf{T}^* + \mathbf{T_C} \quad (13a)$$

$$\mathbf{T} = \mathbf{J}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \quad (13b)$$

$$\mathbf{T_C} = \mathbf{J_C}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C_C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G_C}(\mathbf{q}). \quad (13c)$$

Equation (13) means that the joint actuating n -vector torque $\mathbf{T}$ prior to the compensation procedure is, in fact, decomposed in the two terms:

- The n-vector torque $\mathbf{T}^*$ supplied by the actuators after implementing the compensation
- The n-vector torque $\mathbf{T_C}$ that is supplied by the compensation structure.

The block diagram represented in Fig. 7 corresponds to (13) and shows that a robot follows precisely a given trajectory $\ddot{\mathbf{q}}(t)$ without requiring any torque $\mathbf{T}^*$ from the actuators, if and only if:

$$\mathbf{J_C}(\mathbf{q}) = \mathbf{J}(\mathbf{q}) \quad (14a)$$

$$\mathbf{C_C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \quad (14b)$$

$$\mathbf{G_C}(\mathbf{q}) = \mathbf{G}(\mathbf{q}). \quad (14c)$$

These equations are complex and difficult to match using mechanical structures. An alternative strategy that overcomes those limitations is the use of a software (i.e., programmable) compensation. With the software approach the gravitational terms can be exactly matched. As a result of such cancellation we get TF's with two poles at the origin of the s -plane. Moreover, the configuration-dependent inertial terms can be perfectly compensated without introducing any extra constant ones. Consequently, the total system behaves like a linear double integrator system with acceleration inputs.

As shown in section two, in a stability perspective, (14a) and (14c) compensate terms of first order influence while (14b) compensates terms of second order influence. This means that the Coriolis/centripetal compensation has no influence upon stability; nevertheless, the term $\mathbf{C_C}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ works like a perturbation which has a detrimental effect on the system performance, namely at high velocities. Therefore, it is desirable to have a full compensation according to (14).

In practice software compensation is imperfect because:

- The high computational load required by the algorithm leads to a finite sampling frequency [33], [34];
- Models are ideal being difficult to take into account all the phenomena involved. In this sense, uncompensated effects such as nonlinear friction and backlash will degrade the system performance;
- The real-time estimation of payload parameters is difficult [35]. As condition (14) depends on the payload mass, inaccurate estimations imply imperfect compensation.

In conclusion, software compensation has advantages over hardware compensation but, in practice, it has still limitations. In order to overcome these problems, in the next section, we shall study several complementary control architectures.

IX. On theNonlinear Model- Based Control of Robot Manipulators

In the previous sections we analysed both hardware and software dynamic compensation techniques for robot manipulators. However, these methods have limitations which make difficult the development of a satisfactory practical implementation. Therefore, in order to have good performances, namely stable and accurate trajectory responses, we need to devise complementary control strategies. In this section we shall study two nonlinear model-based controllers. These algorithms have already been proposed [38]–[45] but, due to the highly nonlinear nature of the dynamic phenomena, they are still in a research stage. Based on our previous approach we shall demonstrate that such nonlinear algorithms are, in fact, implementations of dynamic compensation with complementary control structures.

The direct design controller [Fig. 8(a)] adopts the classical position/velocity linear feedback loops together with the dynamic compensation. For this system we have:

$$\mathbf{T}^* = \mathbf{0} \tag{15a}$$

$$\ddot{\mathbf{q}}(t) = \mathbf{D}\mathbf{q_d}(t) - [\mathbf{K_1}\dot{\mathbf{q}}(t) + \mathbf{K_2}\mathbf{q}(t)] \tag{15b}$$

If $\mathbf{D}, \mathbf{K}_1$, and $\mathbf{K}_2$ are diagonal matrices (i.e., $\mathbf{D} = \mathrm{Diag}(d_i), \mathbf{K}_1 = \mathrm{Diag}(K_{1i})$ and $\mathbf{K}_2 = \mathrm{Diag}(K_{2i}), i = 1, \cdots n$) then, for a perfect compensation, we have decoupled outputs with TF's given by:

$$\frac{Q_i(s)}{Q_{di}(s)} = \frac{d_i}{s^2 + K_{1i}s + K_{2i}}. \tag{16}$$

For adequate values of d_i, K_{1i}, and K_{2i} the system has a stable second order response; moreover, for a unity gain at low frequencies we must have $d_i = K_{2i}$. However, we can get superior performances by modifying the control architecture. Fig. 8(b) presents the block diagram of an alternative architecture. This controller (computed torque method) redefines the position and velocity feedback loops according to the equations:

$$\mathbf{T}^* = \mathbf{0} \tag{17a}$$

$$\ddot{\mathbf{q}}(t) = \ddot{\mathbf{q}}_\mathbf{d}(t) + \mathbf{K_1}[\dot{\mathbf{q}}_\mathbf{d}(t) - \dot{\mathbf{q}}(t)] + \mathbf{K_2}[\mathbf{q_d}(t) - \mathbf{q}(t)] \tag{17b}$$

For a perfect compensation we have TFs such as:

$$\frac{Q_i(s)}{Q_{di}(s)} = \frac{s^2 + K_{1i}s + K_{2i}}{s^2 + K_{1i}s + K_{2i}} = 1 \tag{18}$$

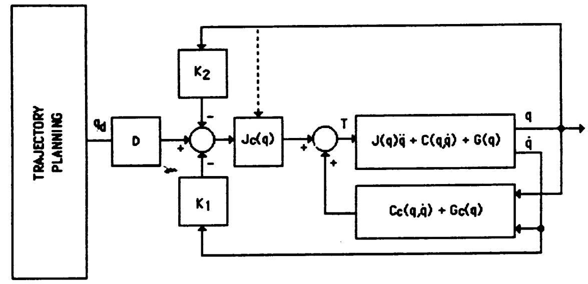
and therefore, we have a system that reveals an ideal path tracking capability. Unfortunately, this ideal situation is not possible in practice and any mismatch between $\mathbf{T}$ and $\mathbf{T_C}$ will degrade the system performance. Considering the real situation, having imperfect compensation, we have for the direct design controller:

$$\mathbf{D}\mathbf{q_d}(t) - [\ddot{\mathbf{q}}(t) + \mathbf{K_1}\dot{\mathbf{q}}(t) + \mathbf{K_2}\mathbf{q}(t)] = \delta\mathbf{T} \tag{19}$$

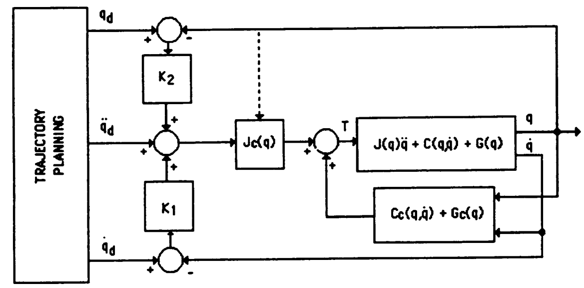
while, for the computed torque algorithm, it comes:

$$\ddot{\mathbf{e}} + \mathbf{K_1}\dot{\mathbf{e}} + , \mathbf{K_2}\mathbf{e} = \delta\mathbf{T} \tag{20a}$$

$$\mathbf{e} = \mathbf{q_d} - \mathbf{q} \tag{20b}$$



(a)



(b)

Fig. 8. Nonlinear model-based controllers. (a) Direct design algorithm. (b) Computed torque algorithm.

where $\delta\mathbf{T}$ is a measure of the compensation error given by the expression:

$$\delta\mathbf{T} = \mathbf{J_C}(\mathbf{q})^{-1}\{[\mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q})] - [\mathbf{J_C}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C_C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G_C}(\mathbf{q})]\}. \tag{21}$$

These nonlinear equations are complex and, therefore, the effects of the imperfect compensation upon the system performances are difficult to predict.

The classical approach provides the tools for the study of these control systems. In fact, considering $P(s)$ the perturbation signal that corresponds to $\delta\mathbf{T}$, then (Fig. 9) the system responses become:

$$Q_i(s) = \frac{d_i}{s^2 + K_{1i}s + K_{2i}} Q_{di}(s) + \frac{P(s)}{s^2 + K_{1i}s + K_{2i}} \tag{22a}$$

$$Q_i(s) = Q_{di}(s) + \frac{P(s)}{s^2 + K_{1i}s + K_{2i}} \tag{22b}$$

for the direct design and the computed torque, respectively. Therefore, for both algorithms, the closed loop poles are the roots of the polynomial $s^2 + K_{1i}s + K_{2i}$ and the effects of $P(s)$ are given by the second term in (22). Furthermore, in what concerns stability, we can study the open loop TF:

$$\frac{K_{1i}s + K_{2i}}{s^2} \tag{23}$$

using the standard methods for the analysis of linear systems.

In conclusion, classical system theory can, in fact, be applied as a design tool in order to analyse and develop nonlinear model-based controllers for robot manipulators.

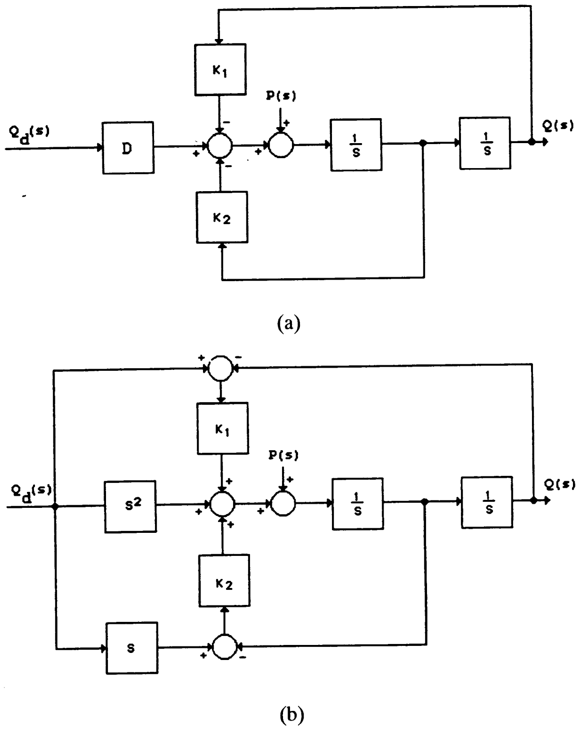


Fig. 9. Linear model of the control systems. (a) Direct design algorithm. (b) Computed torque algorithm.

X. Conclusion

We have presented a classical, Laplace-based, perspective of the dynamics of robot manipulators. It was shown that manipulators can be open-loop unstable and, therefore, stabilizing techniques are required. Present day controllers for industrial manipulators implement a PID scheme and a position/velocity feedback on each robot joint. Such structure is simple and may achieve stability, however, it does not provide either output decoupling or satisfactory performance in the entire operational space. Furthermore, the adjustment of their (interacting) parameters is an overwhelming task given the absence of systematic procedures.

The compensation of the robot dynamics may lead to systems having better performance. In this line of thought, hardware and software compensation techniques are analysed. Software compensation proves to be superior but it has, still, practical limitations because it requires perfect modeling. Therefore, practical systems need not only a software compensation but also additional control actions. Algorithms such as the direct design and the computed torque methods implement this philosophy. These controllers have nonlinear loops yet, their study in the proposed perspective showed that a linear analysis was possible. Therefore, classical design tools can be adopted in order to develop real implementations.

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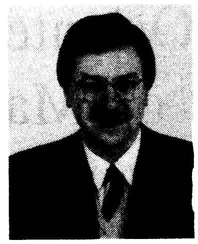

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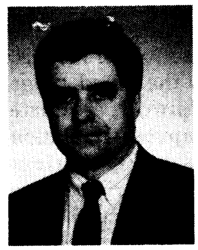

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