Proceedings of ICCC 2003
IEEE International Conference on Computational Cybernetics
Siófok, Hungary, August 29 – 31, 2003

Fractional-Order Position/Force Control of Two Cooperating Manipulators

N. M. Fonseca Ferreira
Institute of Engineering of Coimbra
Coimbra Polytechnic
Dept. of Electrical Engineering, Rua Pedro Nunes - Quinta da Nora 3030-199 COIMBRA
Portugal
nunomig@isec.pt

J. A. Tenreiro Machado
Institute of Engineering of Porto
Porto Polytechnic
Dept. of Electrical Engineering, Rua Dr. António Bernardino de Almeida 431 4200-072 PORTO
Portugal
jtm@dee.isep.ipp.pt

Abstract – **In this paper it is studied the implementation of fractional-order algorithms in the position/force control of two cooperating robotic manipulators. The system performance is analyzed in terms of time and frequency response for different operating conditions.**

I. INTRODUCTION

Two robots carrying a common object are a logical alternative for the case in which a single robot is not able to handle the load [1 - 2]. Nevertheless, with two cooperative robots the resulting interaction forces have to be accommodated rather than rejected. Consequently, in addition to position feedback, force control is also required to accomplish the task [3 - 4].

There are two basic methods for force control, namely the hybrid position/force and the impedance schemes. The first algorithm was proposed by Raibert [4] and separates the task into two orthogonal subspaces corresponding to the force and the position subspaces. Once established the subspace decomposition two independent controllers are designed. The second algorithm was first proposed by Hogan [1]. In this method, by a proper choice of the arm impedance the interaction forces can be controlled to obtain an adequate response.

This paper studies the position/force control of two cooperative manipulators, carrying a common load, using fractional-order (*FO*) controllers [6 - 8].

In this line of thought the paper is organized as follows. Section two develops a *FO* algorithm for the position/force control of two robotic arms. Section three presents several experiments for the system performance evaluation. Finally, section four outlines the main conclusions.

II. POSITION FORCE CONTROL OF TWO ARMS

When two robots grasp an object (Fig. 1), and move it from one location to another, a coordinated motion is required. In order to get good performances it is necessary to specify no only the desired motion of each robot but also the corresponding handling force.

In the system under study the contact of the robot gripper with the load is modeled through a linear system with a mass M, a damping B and a stiffness K.

On the other hand, the dynamics of a robot with n links interacting with the environment is modeled as:

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) - \mathbf{J}^{\mathbf{T}}(\mathbf{q})\mathbf{F} \tag{1}$$

where $\boldsymbol{\tau}$ is the $n \times 1$ vector of actuator torques, $\mathbf{q}$ is the $n \times 1$ vector of joint coordinates, $\mathbf{H(q)}$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the $n \times 1$ vector of centrifugal/Coriolis terms and $\mathbf{G(q)}$ is the $n \times 1$ vector of gravitational effects. The $n \times m$ matrix $\mathbf{J^T(q)}$ is the transpose of the Jacobian matrix of the robot and $\mathbf{F}$ is the $m \times 1$ vector of the force that the (m-dimensional) environment exerts in the robot gripper.

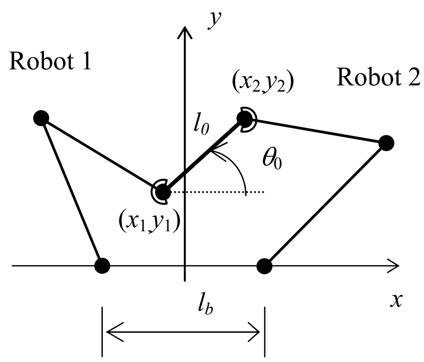

Fig. 1. Two $2R$ cooperating robots for the manipulation of an object with length l_0 and orientation θ_0.

We consider $2R$ manipulators with dynamics given by:

$$\mathbf{H}(\mathbf{q}) = \left[\begin{array}{c|c} (m_1+m_2)r_1^2 + m_2 r_2^2 + & m_2 r_2^2 + \\ 2m_2 r_1 r_2 C_2 + J_{1m} + J_{1g} & m_2 r_1 r_2 C_2 \\ \hline m_2 r_2^2 + m_2 r_1 r_2 C_2 & m_2 r_2^2 + \\ & J_{2m} + J_{2g} \end{array}\right] \tag{2a}$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} -m_2 r_1 r_2 S_2 \dot{q}_2^2 - 2m_2 r_1 r_2 S_2 \dot{q}_1 \dot{q}_2 \\ m_2 r_1 r_2 S_2 \dot{q}_1^2 \end{bmatrix} \tag{2b}$$

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} g(m_1 r_1 C_1 + m_2 r_1 C_1 + m_2 r_2 C_{12}) \\ g m_2 r_2 C_{12} \end{bmatrix} \tag{2c}$$

$$\mathbf{J^T}(\mathbf{q}) = \begin{bmatrix} -r_1 S_1 - r_2 S_{12} & r_1 C_{11} + r_2 C_{12} \\ -r_2 S_{12} & r_2 C_{12} \end{bmatrix} \tag{2d}$$

where $C_{ij} = \cos(q_i + q_j)$ and $S_{ij} = \sin(q_i + q_j)$.

The numerical values adopted for the $2R$ robots and the object are $m_1 = 0.5\ kg$, $m_2 = 6.25\ kg$, $r_1 = 1.0\ m$, $r_2 = 0.8\ m$, $J_{1m} = J_{2m} = 1.0\ kgm^2$, $J_{1m} = J_{2m} = 4.0\ kgm^2$ $l_b = l_0 = 1.0\ m$ and $\theta_0 = 0\ deg$, $B_1 = B_2 = 1\ Ns/m$ and $K_1 = K_2 = 10^4\ N/m$.

The controller architecture (Fig. 3) is inspired on the impedance and compliance schemes. Therefore, we establish a cascade of force and position algorithms as internal an external feedback loops, respectively, where x_d and F_d are the payload desired position coordinates and contact forces.

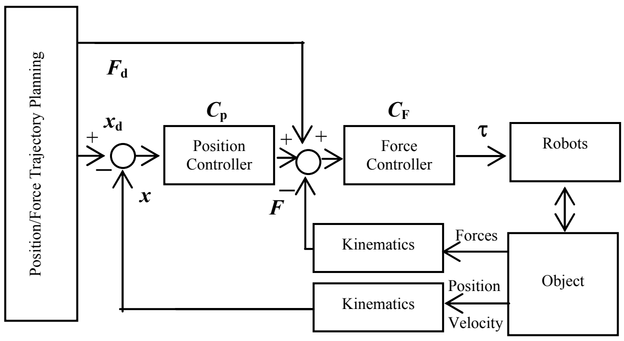

Fig. 2. The position/force controller.

We adopt *FO* algorithms, both at the position and force control loops, namely of the type $C(s)=K_0 s^{\alpha}$ with gain K_0 and fractional order $-1 < \alpha < 1$. The corresponding discrete – time approximations $C(z)$ are implemented through the 4th–order Pade equations:

$$C_P(s) = K_{P0} s^{\alpha_P} \Rightarrow C_P(z) \approx K_P \frac{a_0 z^n + ... + a_n}{b_0 z^n + ... + b_n} \quad (3a)$$

$$C_F(s) = K_{F0} s^{\alpha_F} \Rightarrow C_F(z) \approx K_F \frac{c_0 z^n + ... + c_n}{d_0 z^n + ... + d_n} \quad (3b)$$

where K_P and K_F are the position/force loop gains and a_i, b_i, c_i, d_i, $\in \Re$.

The *FO* controllers were tuned by trial and error, establishing a compromise between fast transients and large overshoots leading to the parameters $K_{Pi} = 10^4$, $K_{Fi} = 10^4$, $\alpha_{Pi} = ½$, $\alpha_{Fi} = -½$, $(i = 1, 2)$.

III. CONTROLLER PERFORMANCES

This section analyses the system performance both in the time and the frequency domains.

A) TIME RESPONSE

We consider different working situations in order to study the effect of changing the payload mass and the contact surface between the gripper and the object. Therefore, the parameters M, B_i and K_i $(i = 1, 2)$ are varied shown in Table I.

In all experiments the controller sampling frequency is $f_c = 10$ kHz for the operating point A ≡ $\{x,y\} \equiv \{1,1\}$ of the object and a contact force of each gripper of $\{Fx_i, Fy_i\} \equiv \{0.5, 5\}$ Nm.

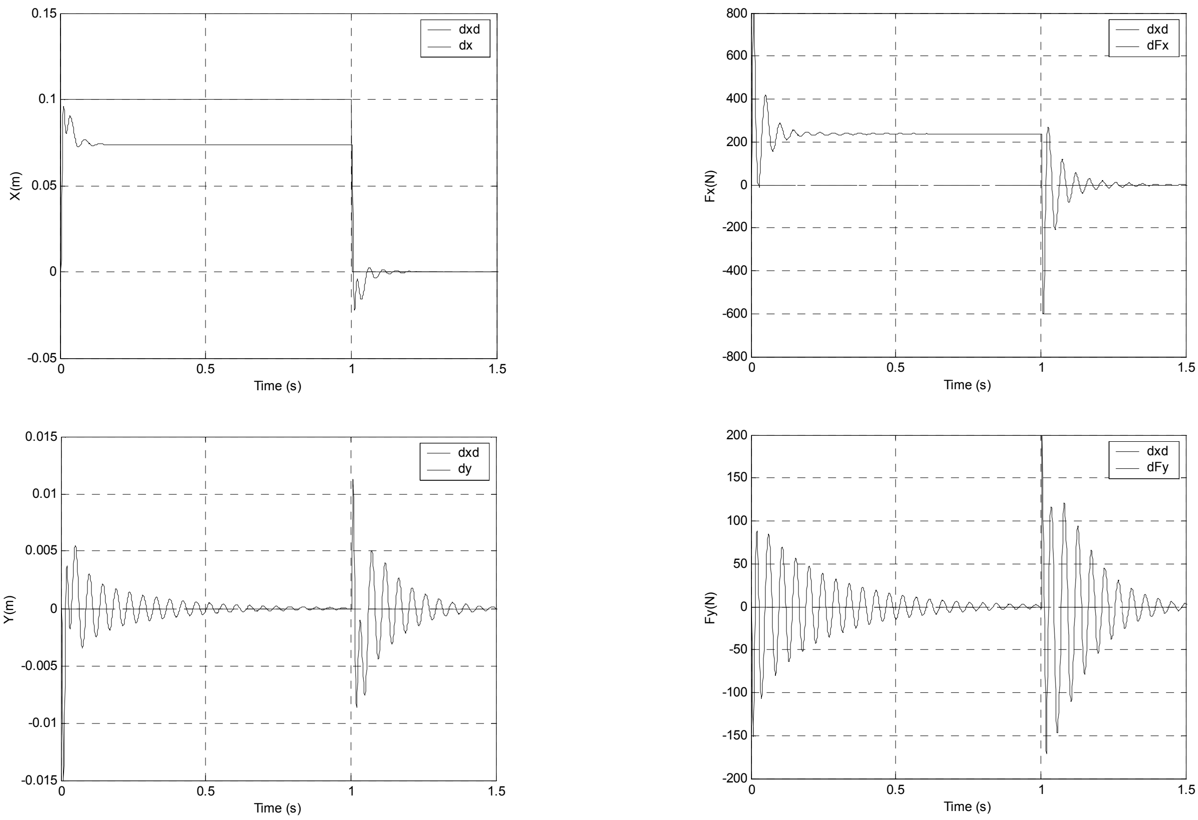

Fig. 3. Time response $\delta x(t)$, $\delta y(t)$, $\delta Fx(t)$, $\delta Fy(t)$ for a reference position perturbation $\delta x_d = 0.1$ m and a payload with the parameters of case S2.

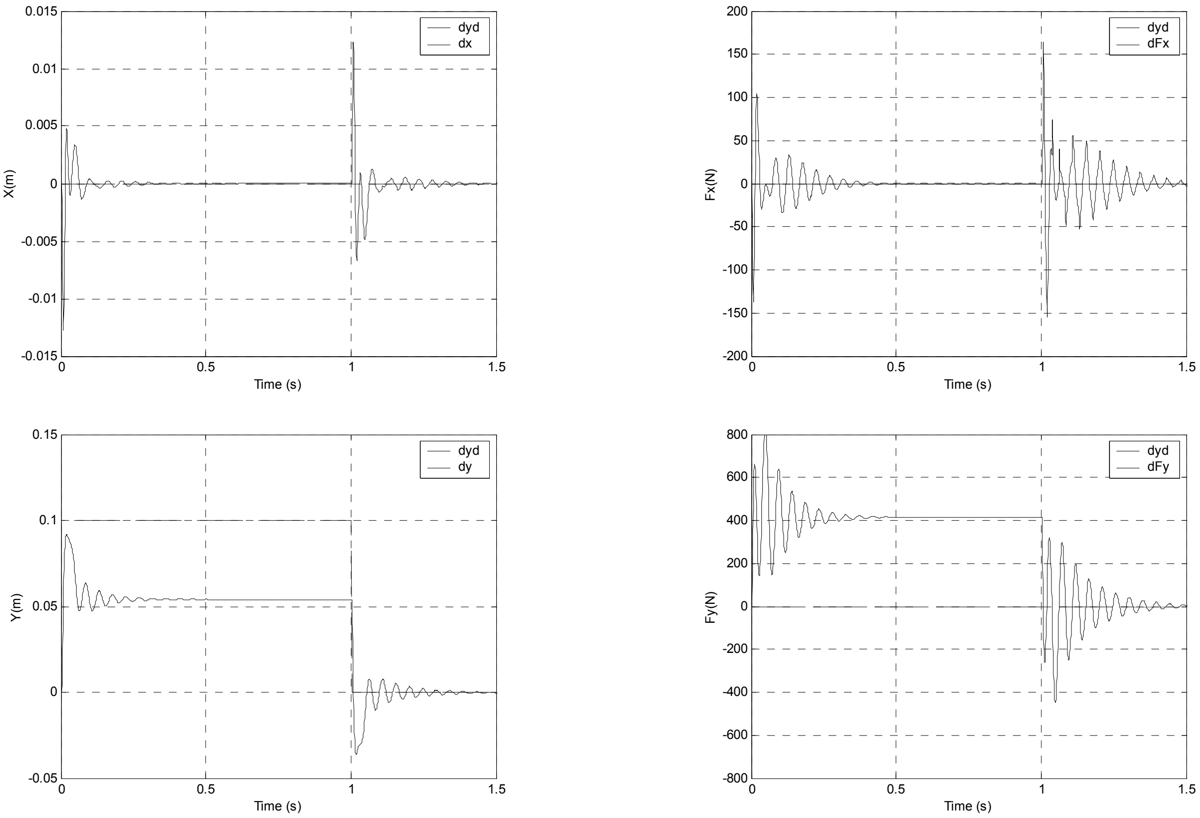

Fig. 4. Time response δ$x(t)$, δ$y(t)$, δ$Fx(t)$, δ$Fy(t)$ for a reference position perturbation δy_d = 0.1 m and a payload the parameters of case S2.

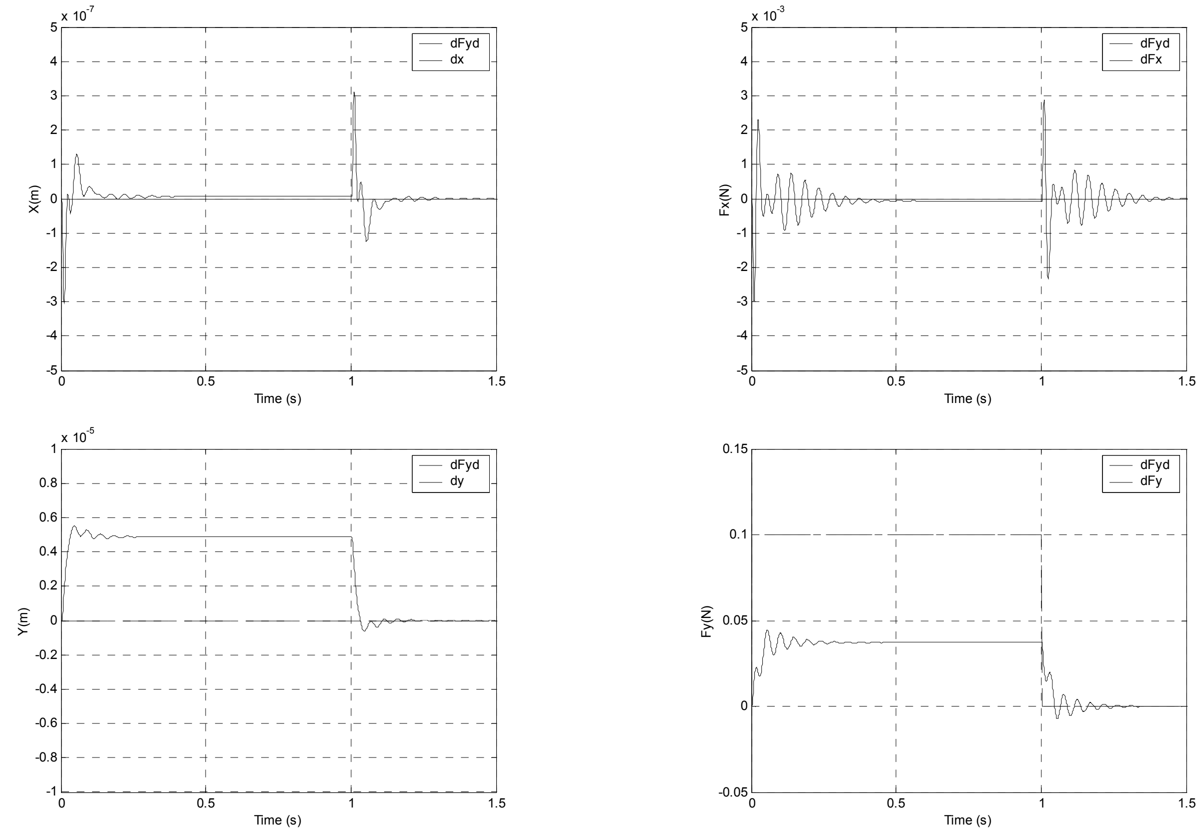

Fig. 5. Time response δ$x(t)$, δ$y(t)$, δ$Fx(t)$, δ$Fy(t)$ for a reference force perturbation δFy_d = 0.1 N and a payload with the parameters of case S2

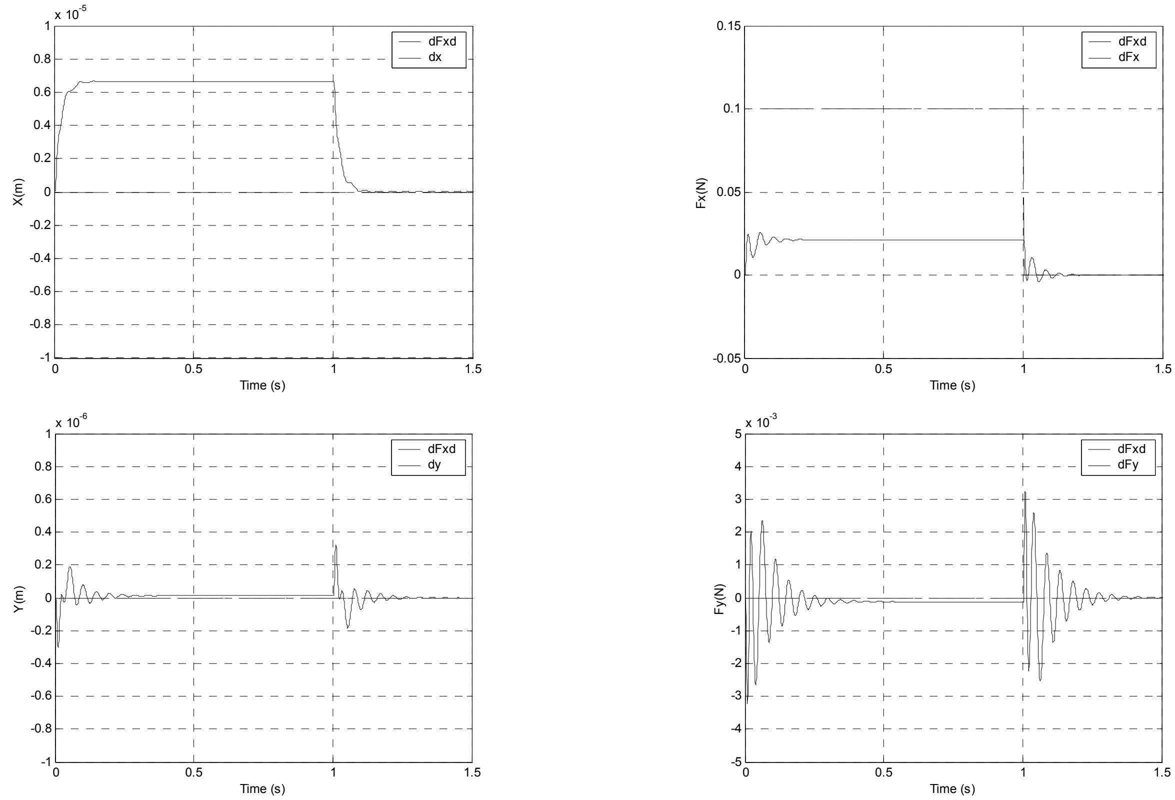

Fig. 6. Time response $\delta x(t)$, $\delta y(t)$, $\delta Fx(t)$, $\delta Fy(t)$ for a reference force perturbation $\delta Fx_d = 0.1$ N and a payload with the parameters of case S2.

Bearing this fact in mind, we introduce, separately, stepwise perturbations in the position and force references of the robot 1, namely $\delta x_d = 0.1$ m, $\delta y_d = 0.1$ m, $\delta Fx_d = 0.1$ N and $\delta Fy_d = 0.1$ N, at $t = 1$ sec with a duration $\delta t = 1$ sec.

Figures 3-6 shows the resulting perturbations at the different outputs for the case S2 and Figure 7 shows the time responses for the different load parameters of Table 1.

It is clear that positioning errors increase with the contact friction B_i. Furthermore, the larger the contact stiffness K_i the smaller the position overshoot. On the other hand, the force response presents a large peak for high values of K_i/B_i, revealing differential-like behaviors.

TABLE I – The 2*R* robot parameters.

Simulation	*M* (*kg*)	B_i (*Ns/m*)	K_i (*N/m*)
S1	1	10	10^3
S2	1	10	10^4
S3	1	10^2	10^3
S4	1	10^2	10^4
S5	2	10	10^3
S6	2	10	10^4
S7	2	10^2	10^3
S8	2	10^2	10^4

B) FREQUENCY RESPONSE

Based on the time response to small perturbations at the position and force references we can establish the frequency response, corresponding to linearized transfer functions around the operating point A.

Figure 8 show the frequency response of the robot 1 position and force, namely $|X(\omega)/X_d(\omega)|$, $|Y(\omega)/Y_d(\omega)|$, $|Fx(\omega)/Fx_d(\omega)|$ and $|Fy(\omega)/Fy_d(\omega)|$. Furthermore, the charts reveal that we have a limited variation for a dramatic change on the load parameters. Therefore, we conclude that the *FO* algorithms reveal au adequate stability and a good robustness.

IV. CONCLUSIONS

This paper studied the position/force control of two robots working in cooperation using a fractional-order control algorithm. The dynamic performance of two arms holding an object was analyzed both in the time and the frequency domains. The results revealed that the fractional-order algorithm has a good performance and a high robustness.

V. REFERENCES

[1] N. Hogan, “Impedance control: An Approach to Manipulation, Parts I-Theory, II-Implementation, III-Applications”, ASME J. of Dynamic Systems, Measurement and Control, vol. 107, No. 1, pp. 1-24, 1985.

[2] E. Whitney, “Historical Perspective and State of the Art in Robot Force Control”, IEEE Conf. on Robotics and Automation, St Louis, 1985.

[3] B. Siciliano and L. Villani, “Robot Force Control”, Kluwer Academic Publishers, 1999.

[4] M. H. Raibert and J. J. Craig, , “Hybrid Position/Force Control of Manipulatores”, ASME J. of Dynamic Systems, Measurement, and Control” vol. 2, No. 2, pp.126-133, vol. 1, 1981.

[5] N. M. Fonseca Ferreira and J. A. Tenreiro

Machado, "Manipulation Analysis of Two Cooperating Arms", 10th IEEE Int. Conf. on Advanced Robotics, Budapest, Hungary, 2001.

[6] A. Oustaloup, La Commande CRONE: Commande Robuste d'Ordre Non Entier, Editions Hermes, 1991.

[7] J. Tenreiro Machado, "Analysis and Design of Fractional-Order Digital Control Systems", *J. of Systems Analysis, Modelling and Simulation*, vol. 27, pp. 107–122, 1997.

[8] I. Podlubny, "Fractional-Order Systems and $PI^{\lambda}D^{\mu}$-Controllers", IEEE Trans. on Automatic Control, vol. 44, No. 1, pp. 208–213, 1999.

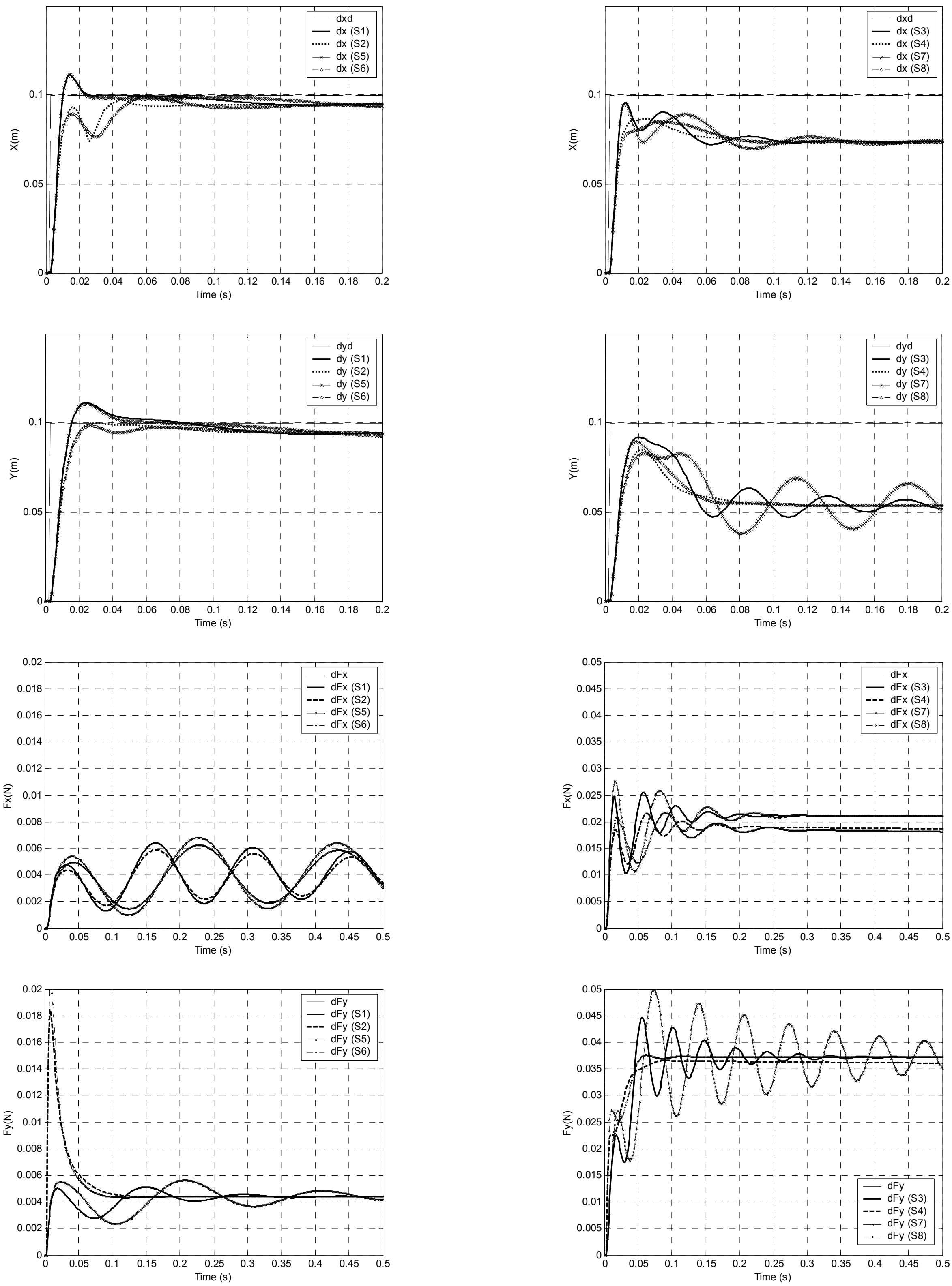

Fig. 7. Time response $\delta x(t)$, $\delta y(t)$, $\delta Fx(t)$ and $\delta Fy(t)$ for reference position perturbations $\delta x_d = 0.1$ m, $\delta y_d = 0.1$ m, $\delta Fx_d = 0.1$ N, $\delta Fy_d = 0.1$ N, and a payload with the different parameters of cases S1 to S8.

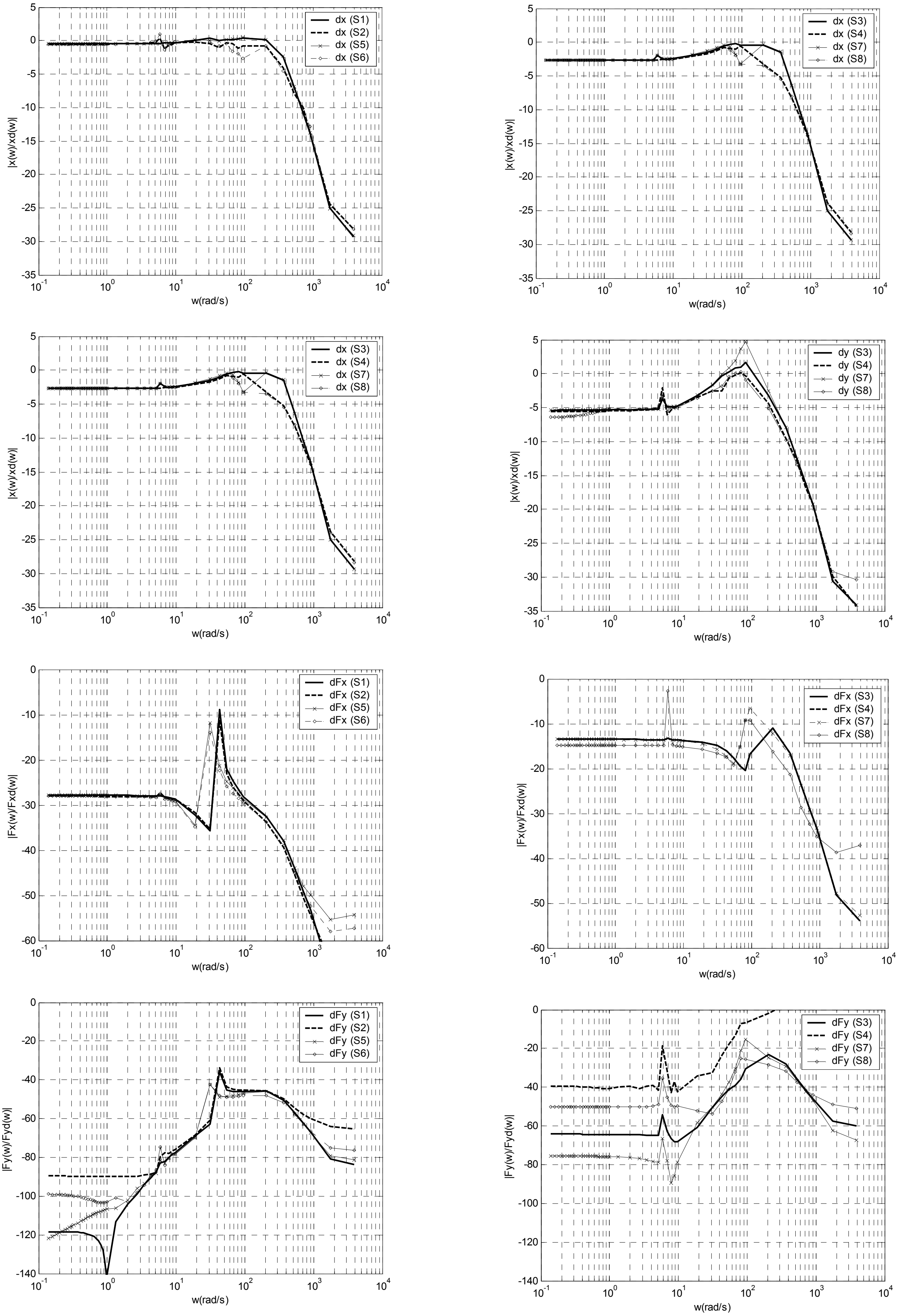

Fig. 8. Frequency response $|X(\omega)/X_d(\omega)|$, $|Y(\omega)/Y_d(\omega)|$, $|Fx(\omega)/Fx_d(\omega)|$ and $|Fy(\omega)/Fy_d(\omega)|$ for reference perturbations $\delta x_d = 0.1$ m, $\delta y_d = 0.1$ m, $\delta Fx_d = 0.1$ N, $\delta Fy_d = 0.1$ N and a payload with the different parameters of cases S1 to S8.