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# Fractional Order Dynamics in a Genetic Algorithm

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**Abstract**

*This work addresses the fractional-order dynamics during the evolution of a Genetic Algorithm population (GA) for generating a robot manipulator trajectory. The GA objective is to minimize the trajectory space/time ripple without exceeding the torque requirements. In order to investigate the phenomena involved in the GA population evolution, the mutation is exposed to excitation perturbations and the corresponding fitness variations are evaluated. The input/output signals are studied revealing a fractional-order dynamic evolution, characteristic of a long-term system memory.*

**Keywords:** Fractional Calculus, Genetic Algorithms, Robotic Manipulators, Trajectory Planning.

## 1 INTRODUCTION

In the last decade Genetic Algorithms ($GAs$) have been applied in a plethora of fields such as in image processing, pattern recognition, speech recognition, control, system identification, optimization, planning and scheduling [1]. In the area of robotics several $GA$-schemes for trajectory planning were proposed. A possible approach consists in adopting the differential inverse kinematics for generating the manipulator trajectories [2, 3]. However, the algorithm must take into account the problem of kinematic singularities that may be hard to tackle. To avoid this problem, other methods for the trajectory generation are based on the direct kinematics [4, 5, 6, 7].

Fractional Calculus ($FC$) stems from the beginning of theory of differential and integral calculus [8, 9]. Nevertheless, the application of $FC$ has been scarce until recently, but the advances in the theory of chaos motivated a renewed interest in this field. In the last two decades we can mention research on viscoelasticity/damping, chaos/fractals, biology, electronics, signal processing, diffusion and wave propagation, percolation, modeling, control and irreversibility [10, 11, 12, 13, 14, 15, 16] .

Bearing these ideas in mind, this paper analyzes the fractional-order dynamics in the population of a GA-based trajectory planning scheme for mechanical manipulators. The article is organized as follows. Section 2 introduces the problem, the $GA$ method for its resolution and a run-out experiment, respectively. Based on this formulation, section 3 presents the results for several simulations involving different excitation conditions and studies the resultant signals and dynamic phenomena. Finally, section 4 outlines the main conclusions.

## 2 THE GA TRAJECTORY PLANNING SCHEME

This section presents the $GA$ planning scheme to render an optimized trajectory, having a reduced ripple in the space/time evolution, while not exceeding a maximum pre-defined torque. We consider a two-link manipulator, that is required to move between two points in the workspace, and a $GA$ that uses the direct kinematics to avoid singularity problems.

### 2.1 Trajectory Representation

The manipulator can move between two points of the workspace. Therefore, the initial and final configurations are given by the inverse kinematic equations. The path is encoded directly, using real codification, as strings in the joint space to be used by the $GA$ as:

$$[\Delta t, (q_{11}, q_{21}), \ldots, (q_{1j}, q_{2j}), \ldots, (q_{1m}, q_{2m})] \qquad (1)$$

The $i$th joint variable for a robot intermediate $j$th position is $q_{ij}$, the chromosome is constituted by $m$ genes (configurations) and each gene has two values. The joint variables $q_{ij}$ are initialized in the range $]-180^o,+180^o]$. It is important to note that the initial and final configurations have not been encoded into the string because this configuration remains unchanged throughout the trajectory search. Moreover, the additional parameter $\Delta t$ is introduced in the chromosome to specify the time between two consecutive configurations.

### 2.2 Operators in Genetic Algorithm

The initial population of strings is generated at random and the search is then carried out among this population. The evolution of the population elements is non-generational, meaning that the new replace the worst elements. The main different operators adopted in the *GA* are reproduction, crossover and mutation.

In what concerns the reproduction operator, the successive generations of new strings are generated based on their fitness values. In this case, it is used a 5-tournament [17] to select the strings for reproduction. Furthermore, it is used another 5-tournament selection to choose the strings to be replaced by the children strings.

For the crossover operator it is adopted the single point technique and, therefore, the crossover point is only allowed between genes or, by other words, the crossover operator can not disrupt genes.

The mutation operator replaces one gene value $x_t$ with a given probability $p_m$. The new value $x_{t+1}$ is obtained by the equation $x_{t+1} = x_t \pm N[0,(2\pi)^{-1/2}]$, where $N$ represents the Normal probability distribution.

### 2.3 Evolution criteria

Five indices are used to qualify the evolving trajectory robotic manipulators. All indices are translated into penalty functions to be minimized. Each index is computed individually and is integrated in the fitness function evaluation.

The fitness function $f$ adopted for evaluating the candidate trajectories is defined as:

$$f = \beta_1 f_{ot} + \beta_2 \dot{q} + \beta_3 \ddot{q} + \beta_4 \dot{p} + \beta_5 \ddot{p} \tag{2a}$$

$$f_{ot} = \sum_{j=1}^{m} \left( f_1^j + f_2^j \right) \tag{2b}$$

$$f_i^j = \begin{cases} 0 & \text{if } \left|\tau_i^j\right| < \tau_{i\ \max} \\ \left|\tau_i^j\right| - \tau_{i\ \max} & \text{otherwise} \end{cases} \tag{2c}$$

$$\dot{q} = \sum_{j=1}^{m} \sum_{i=1}^{2} \dot{q}_{ij}^2 \tag{2d}$$

$$\ddot{q} = \sum_{j=1}^{m} \sum_{i=1}^{2} \ddot{q}_{ij}^2 \tag{2e}$$

$$\dot{p} = \sum_{j=2}^{m} d\left(p_j,\ p_{j-1}\right)^2 \tag{2f}$$

$$\ddot{p} = \sum_{j=3}^{m} \left| d\left(p_j,\ p_{j-1}\right) - d\left(p_{j-1},\ p_{j-2}\right) \right|^2 \tag{2g}$$

The indices $f_{ot}$, $\dot{q}$, $\ddot{q}$, $\dot{p}$, $\ddot{p}$ are discussed in the sequel. The optimization goal consists in finding a set of design parameters that minimize $f$ according to the priorities given by the weighting factors $\beta_i$ ($i$ = 1,.., 5). The $f_{ot}$ index represents the amount of excessive driving, in relation to the maximum torque $\tau_{imax}$, that is demanded for the $i$th joint motor for the trajectory under consideration. The joint velocities (2d) are used to minimize the manipulator traveling distance. This equation is used to optimize the traveling distance because, if the curve length is minimized, the ripple in the space trajectory is indirectly reduced. For a function $y = g(x)$ the distance curve length is $\int[1 + (dg/dt)^2]\ dx$ and, consequently, to minimize the distance curve length it is adopted the simplified expression $\int(dg/dt)^2\ dx$. The fitness function maintains the quadratic terms so that the robot configurations are uniformly distributed between the initial and final configurations. The joint accelerations (2e) are used to minimize the ripple in the time evolution of the robot trajectory. The cartesian velocities (2f) minimize the total trajectory length, from the initial point up to the final point, where $p_j$ is the robot $j$ intermediate arm Cartesian position and $d(\cdot,\cdot)$ is a function that gives the distance between the two arguments. Finally, the cartesian acceleration (2g) is responsible for reducing the ripple in time evolution of the arm velocities.

### 2.4 Simulation results

In order to evaluate the performance of the *GA* planner, in this subsection we consider a simple experiment consisting on moving a robotic arm from the starting point A≡{1.25, –0.30} up to the final point B≡{–0.50, 1.40}. In the *GA* are adopted crossover and mutation probabilities $p_c = 0.8$ and $p_m = 0.05$, respectively, a population of 200 elements for the intermediate arm configurations, a string size of $m = 7$ and a 5-tournament selection scheme. The robot links have a length of $l_i = 1$ m and a mass of $m_i = 1$ kg ($i$ = 1,2). The joints 1 and 2 that are free to rotate $360^o$ and the maximum allowed torques are $\tau_{1max} = 16$ Nm and $\tau_{2max} = 5$ Nm, respectively. The time between two consecutive configurations is restricted to the interval $0.05 \leq \Delta t \leq 1.60$ sec.

For simplicity, the normalized time between two consecutive *GA* generations is considered $T = 1$ sec, without losing generality, because it is always possible to perform a time re-scaling.

Figures 1 to 4 show the manipulator trajectories in the $\{x,y\}$ plane and the joint positions, velocities and torques, respectively. Figure 5 depicts the percentiles $P_n$, $n = \{0, 30, 70, 100\}$%, of the *GA*-population fitness during the evolution.

The trajectory presents a smooth behavior, both in the space and time evolution and the required joint torques do not exceed the imposed limitations.

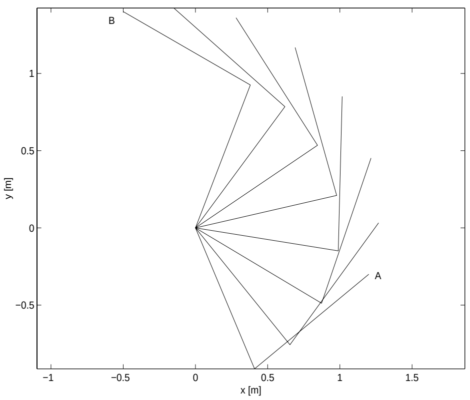


***Figure 1:*** *Robot trajectory in the* $\{x,y\}$ *plane*

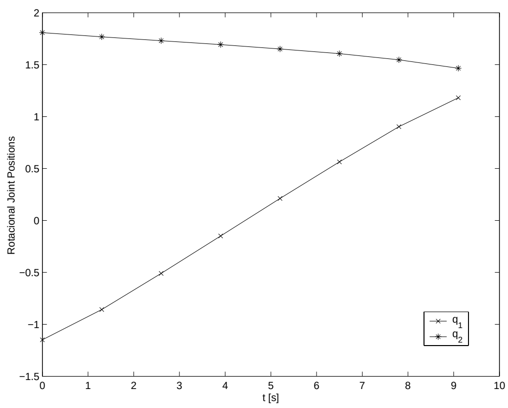


***Figure 2:*** *Robot joint positions vs. time t*

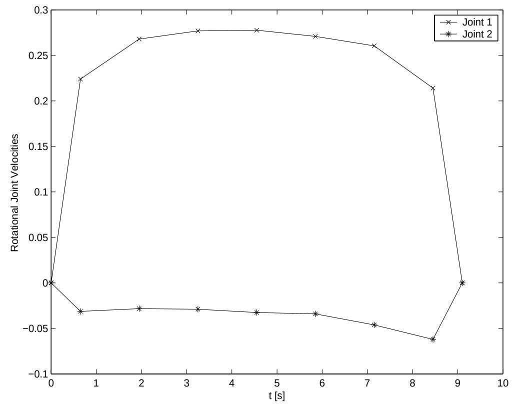


***Figure 3:*** *Robot joint positions vs. time t*

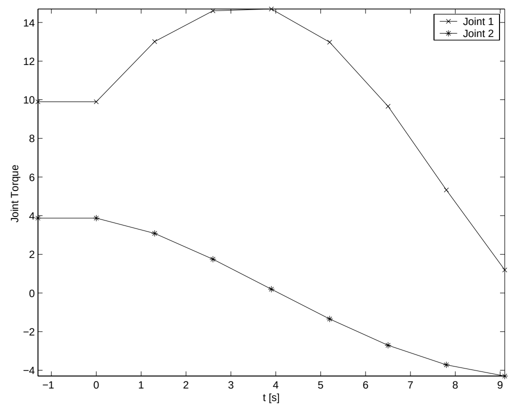


***Figure 4:*** *Robot torques vs. time t*

## 3 EVOLUTION AND FRACTIONAL-ORDER DYNAMICS

This section develops studies the dynamical phenomena involved in the *GA* population. In this perspective, small amplitude perturbations are superimposed over biasing signals of the *G*A system and its influence on the population fitness is evaluated. The experiments reveal a fractional-order dynamics with characteristics with resemblances of those appearing in many chaotic systems.

The *GA* system is stimulated by perturbing the mutation probability through a white noise signal and the corresponding modification of the population fitness is evaluated. Therefore, the variation of the mutation probability and the resulting fitness modification of the *GA* population, during the evolution, can be viewed as the system input and output signals versus time, respectively.

The excitation signal has small amplitude and 'acts' upon the *GA*-system during a time period $T_{exc}$. In this line of thought, a white noise signal $\Delta p$ is added to the mutation probability $p_m$ of the joint variables genes and the new mutation probability $p_{m\_noise}$ is calculated by the following formula:

$$p_{m\_noise} = \begin{cases} 0 & \text{if } p_m + \Delta p < 0 \\ 1 & \text{if } p_m + \Delta p > 1 \\ p_m + \Delta p & \text{otherwise} \end{cases} \qquad (3)$$

Consequently, the input signal is the difference between the two cases, that is $\delta p_m(T) = p_{m\_noise}(T) - p_m(T)$. On the other hand, the output signals are the difference in the population fitness $n$-percentiles with and without noise, that is $\delta P_n(T) = P_{n\_noise}(T) - P_n(T)$.

Figures 6 and 7 show the input signal $\delta p_m$, in the generation time and frequency domains, for a $\Delta p = 0.12 p_m$ perturbation in the mutation probability and an excitation period of $T_{exc} = 100$ generations. Figures 8 and 9 show the corresponding output vari-

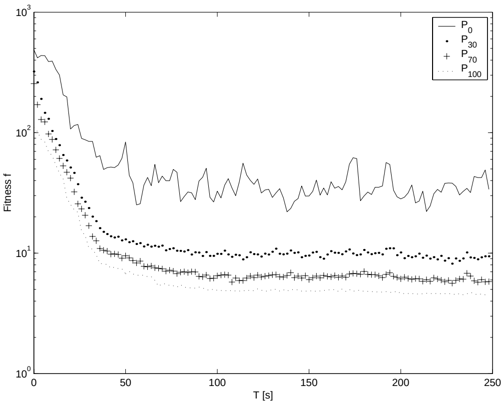


***Figure 5:*** *Percentiles of the population fitness vs. generations T*

ation $\delta P_{50}$, for the percentile $n = 50\%$ of the fitness function. The transfer function $H_n(j\omega)$, between the input and output signals, and the fractional order analytical approximation $G_n(j\omega)$ are depicted in figure 10.

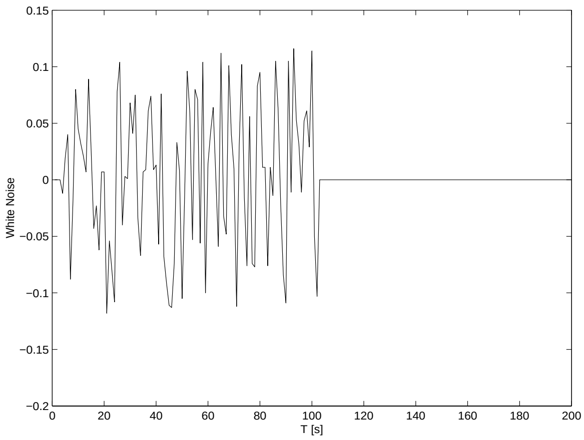


***Figure 6:*** *Input perturbation $\delta p_m(T)$ injected in the mutation probability during $T_{exc}$ = 100 generations*

The numerical data of the system transfer functions are approximated by analytical expressions with gain $k \in \Re$, one zero and one pole $(a,b) \in \Re$ of fractional orders $(\alpha,\beta) \in \Re$, respectively, given by:

$$G_n(s) = k \frac{\left(\frac{s}{a}\right)^{\alpha} + 1}{\left(\frac{s}{b}\right)^{\beta} + 1} \tag{4}$$

A *GA* adopting a real string identifies the $G_n(s)$ parameters using the representation $[k,\ a,\ b,\ \alpha,\ \beta]$.

The main operators are identical to the deployed in section 2.2 but, when one mutation occurs the corresponding value $\{x_1,\ldots,x_5\} \equiv \{k,\ a,\ b,\ \alpha,\ \beta\}$ is changed according with the equations:

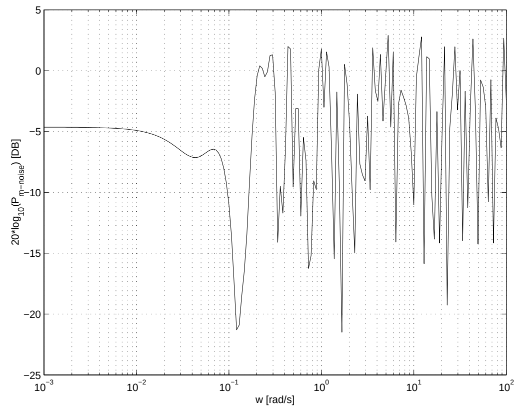


***Figure 7:*** *Fourier spectrum $F\{\delta p_m(T)\}$ of the mutation probability variation*

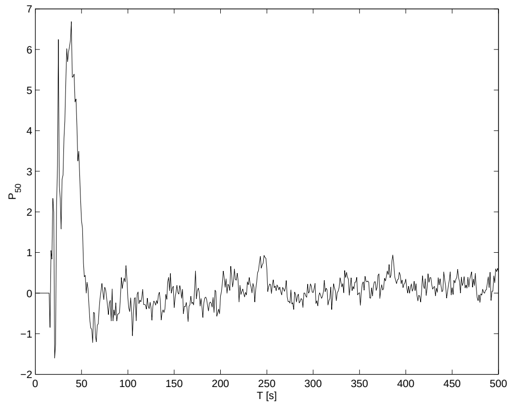


***Figure 8:*** *Output percentile variation $\delta P_{50}(T)$ for an input excitation over $T_{exc}$ = 100 generations*

$$x_{i+1} = 10^{u_i} x_i \tag{5a}$$

$$u_i \sim \mathrm{U}[-\varepsilon_i, +\varepsilon_i] \tag{5b}$$

where $u_i$ is a random number generated through the uniform probability distribution U and $\varepsilon_i$ is fixed according with the range of estimation. In equation (5a) it is adopted an exponential adjusting procedure because the estimation is carried out in a logarithm scale.

The fitness function $f_{n,ide}$ measures, logarithmically, the distance between the numerical $H_n$ and the analytical $G_n$ transfer functions:

$$f_{n,ide} = \sum_{i=1}^{nf} \left[\log_{10} \frac{H_n(\omega_i)}{G_n(\omega_i)}\right]^2 \tag{6}$$

$$G_n(\omega_i) = k \left\{ \frac{\left[\left(\frac{\omega_i}{a}\right)^{\alpha} \mathrm{c}_{\alpha} + 1\right]^2 + \left[\left(\frac{\omega_i}{a}\right)^{\alpha} \mathrm{s}_{\alpha}\right]^2}{\left[\left(\frac{\omega_i}{b}\right)^{\beta} \mathrm{c}_{\beta} + 1\right]^2 + \left[\left(\frac{\omega_i}{b}\right)^{\beta} \mathrm{s}_{\beta}\right]^2} \right\}^{1/2} \tag{7}$$

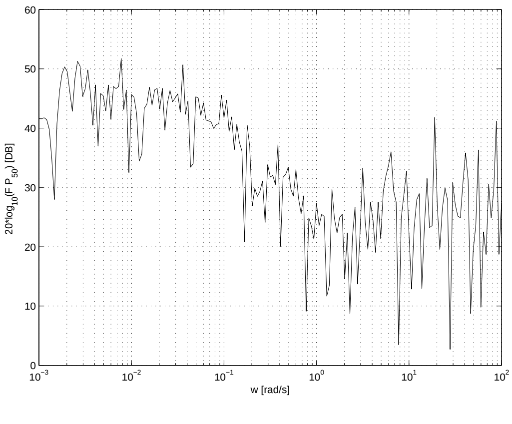


***Figure 9:*** *Fourier spectrum $F\{\delta P_{50}(T)\}$ of the fitness function $n$ = 50% percentile variation*

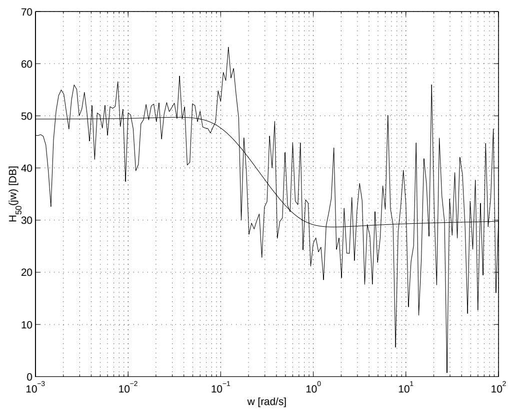


***Figure 10:*** *Transfer function $H_{50}(j\omega) = F\{\delta P_{50}(T)\}/F\{\delta p_m(T)\}$ and the analytical approximation $G_{50}(j\omega)$ for the percentile $n = 50\%$*

where $\mathrm{c}_\alpha = \cos\left(\frac{\pi}{2}\alpha\right)$, $\mathrm{s}_\alpha = \sin\left(\frac{\pi}{2}\alpha\right)$ and *nf* is the total number of sampling points is the frequency domain and $\omega_i$, $i = 1, \ldots, nf$, is the corresponding vector of frequencies.

In order to obtain the parameters of expression (4) are performed several perturbation experiments and the medians of the resulting transfer functions are adopted as the final estimated parameters. Moreover, for evaluating the influence of the excitation period $T_{exc}$ several simulations are developed ranging from $T_{exc} = 20$ to $T_{exc} = 1000$ generations. The relation between the transfer function parameters $\{k, a, b, \alpha, \beta\}$ and $(T_{exc}, P_n)$ can be approximated through a least squares technique leading to the equations:

$$k = 62069\, T_{exc}^{0.852} \mathrm{e}^{-0.011 P_n} \tag{8a}$$

$$a = 0.317\, T_{exc}^{0.204} \mathrm{e}^{-0.0044 P_n} \tag{8b}$$

$$b = 0.012\, T_{exc}^{0.362} \mathrm{e}^{-0.0060 P_n} \tag{8c}$$

$$\alpha = 1.121\, T_{exc}^{-0.005} \mathrm{e}^{0.0019 P_n} \tag{8d}$$

$$\beta = 1.093\, T_{exc}^{-0.028} \mathrm{e}^{0.0025 P_n} \tag{8e}$$

These results reveal that the transfer function parameters have a low dependence on the percentile $P_n$ of the fitness function and, consequently, that the adoption of a particular value for $n$ is of no importance for the study under effect. On the other hand, the period of excitation $T_{exc}$ has a much stronger influence on the parameter variation.

By enabling the zero/pole order to vary freely, we get non-integer values for $\alpha$ and $\beta$, while the adoption of an integer-order transfer function would lead to a larger number of zero/poles to get the same quality in the analytical fitting to the numerical values. The 'requirement' of fractional-order models in opposition with the classical case of integer models is a well-known discussion and even nowadays final conclusions are not clear, since it is always possible to approximate a fractional frequency response through an integer one as long as we make use of a larger number of zeros and poles. Nevertheless, in the present experiments there is a complementary point of view in the direction of *FC*. In fact, analyzing the output signal (Fig. 8) we observe that we have a kind of white noise behavior, with similarities to signals appearing in natural systems, that is auto-sustained, even for time periods very far away from the excitation perturbation period. This characteristic is typical of chaotic systems and suggests further research on the signal dynamics that would occur for other input perturbations, that is, for other *GA* variables and distinct perturbing signal spectra.

## 4 CONCLUSIONS

This paper has analyzed the signal propagation and the dynamic phenomena involved in the time evolution of a population of individuals. The study was established on the basis of a *GA* for the trajectory planning of robot manipulators. While the performance of *GA* schemes has been extensively studied, the influence of perturbation signals over the operating conditions is not well known.

Bearing these ideas in mind, the fundamental aspects of the *FC* calculus were introduced in order to develop approximating transfer functions of variable, either integer or non-integer, order. It was shown that fractional-order models capture phenomena and properties that classical integer-order simply neglect. Moreover, for the case under study the signal evolution have similarities to those revealed by chaotic systems which confirms the requirement for mathematical tools well adapted to the phenomena under investigation. In this line of thought, this article is a step towards the signal and system analysis based on the theory of *FC*.

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