

# Kinematic Aspects of Robotic Biped Locomotion Systems

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## Abstract

*This paper presents the kinematic study of robotic biped locomotion systems. The main purpose is to determine the kinematic characteristics and the system performance during walking. For that objective, the prescribed motion of the biped is completely characterised in terms of five locomotion variables: step length, hip height, maximum hip ripple, maximum foot clearance and link lengths. In this work, we propose three methods to quantitatively measure the performance of the walking robot: locomobility measure, perturbation analysis, and lowpass frequency response. These performance measures are discussed and compared in determining the robustness and effectiveness of the resulting locomotion.*

## 1. Introduction

In recent years a growing field of research in biped locomotion has resulted in a variety of prototypes whose characteristics, in limited sense, resemble the biological systems [1,2,3]. The control of legged vehicles is a difficult and multidisciplinary problem and, even today, is regarded as the most crucial aspect of locomotion [4]. Vukobratovic *et al.* [5] have proposed models and mechanisms to explain biped locomotion. On the other hand, Raibert and his colleagues built hopping and running legged robots in order to study the major issues with dynamic balance [6,7]. In another perspective, Alexandra *et al.* demonstrated [8] the appeal of implementing biological-inspired schemes in the analysis of mechanical systems.

In this line of thought, several researchers are in pursuit of better walking robots and continue to refine their models of locomotion [9,10]. The main objective is to design and control an optimum leg geometry that provides good energy efficiency, the required working volume and simplicity in the structure. This paper concentrates in the kinematic study of a planar biped model with six degrees of freedom and rotational joints. The main purposes are threefold as follows:

- To gain insight into the phenomenon of biped walking.
- To characterise the biped motion in terms of a set of locomotion variables.

- To establish the correlation among these locomotion variables and the system performance.

A wide variety of gait patterns are developed by prescribing the cartesian trajectories of the hip and the lower extremities of the leg. For the analysis of the resulting motion, various kinematic performance measures are proposed and discussed. These kinematic indices can be regarded as quantitative measures of the biped's ability in transporting the section of the body from an initial position to a desired position through the action of the lower limbs. The performance measures and the associated graphical results are used for evaluating the different locomotion patterns.

The remainder of the paper is organised as follows. A short description of the biped model is given in section 2. In section 3 we describe the method used to plan the kinematic trajectories of the biped robot. In section 4 various kinematic performance measures are proposed and discussed mathematically. Given this background, several simulation examples are presented in section 5 to illustrate the application of these methods. Finally, in section 6 we outline the main conclusions and the perspectives for future research.

## 2. Biped model

Fig. 1 shows the planar biped model with examples of the conventions used throughout this paper.

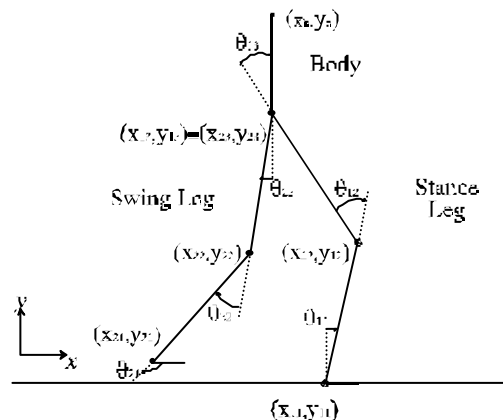


Fig. 1 - Planar biped model.

The kinematic model consists of seven links in order to approximate locomotion characteristics similar to those of the lower extremities of the human body (*i.e.*, body, thigh, shank and foot). In the present study, we consider a planar biped model with seven rigid links interconnected through rotational joints. During locomotion, the stance leg (*i.e.*, the limb that is on the ground) has a degree of freedom formed in the contact between the foot and the ground. A step in the gait of a biped consists of two phases:

- Single support phase in which one leg is in contact with the ground and the other leg swings forward.
- Exchange of support in which the legs trade role.

In the single support phase, the stance leg is in contact with the ground and carries the weight of the body, while the swing leg moves forward in preparation for the next step. The impact of the swing leg is assumed to be perfectly inelastic and the friction is sufficient to ensure that no slippage occurs. Additionally, we consider that the support shifts instantaneously from one limb to the other.

### 3. Forward motion planning

A fundamental problem in robotics is to determine the trajectories that allow the biped robot to walk more skilfully [11]. In early work, the determination of the biped trajectories was made largely on the basis of experience and intuition (*e.g.*, recording kinematic data from human locomotion [12-13]). In this work, the motion planning is accomplished by prescribing the cartesian trajectories of the body and the lower extremities of the leg. Furthermore, the prescribed motion is completely characterised in terms of five locomotion variables. Simultaneously, we determine the relation between the locomotion variables and the trajectories physically admissible. In this context, we introduce the concept of locomotion workspace.

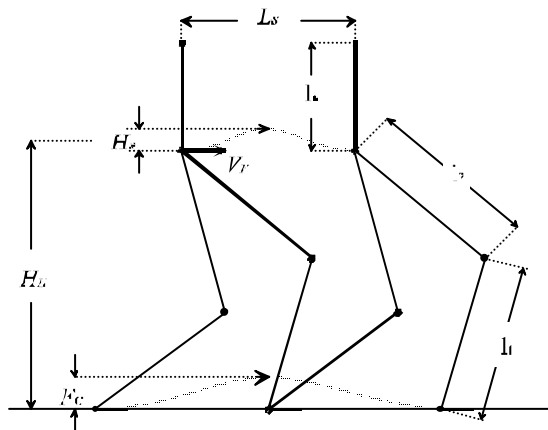


Fig. 2 - Locomotion variables.

### 3.1. Locomotion variables

The prescribed motion of the biped is completely characterised in terms of the following variables: length of the step  $L_s$ , hip height  $H_h$ , maximum hip ripple  $H_r$ , maximum foot clearance  $F_c$  and leg link lengths  $l_1$  and  $l_2$  (Fig. 2). The hip height is defined as the mean height of the hip along the cycle of walking. The hip ripple is measured as the peak-to-peak oscillation magnitude. During the experiments, we examine the role of the link lengths considering that  $l_1 + l_2$  assumes a constant value equal to 1 meter.

### 3.2. Development of the kinematic trajectories

The proposed algorithm accepts the hip and feet's trajectories as inputs and, by means of an inverse kinematics algorithm, generates the related joint trajectories. To improve the smoothness of the motion we impose two additional restrictions: the body maintains an erect posture during locomotion and the forward velocity is constant. Therefore, the horizontal trajectory of the hip is implemented using a constant progression velocity  $V_f$ . One trajectory that undergoes smooth motion is the flat trajectory in which the stance leg adjusts itself so that the hip maintains a constant height. Simultaneously, it is of interest to exploit trajectories in which the hip translates with some vertical motion. In order to simplify the problem, we consider that such motions are produced based on sinusoidal functions.

In dynamic walking, at each footfall, the system may suffer impacts and incurs on additional accelerations that influence the forward velocity [14]. For this reason, we must impose a set of conditions (continuity condition) on the leg velocities so that the feet are placed on the ground while avoiding the impacts. We denote the moment of exchange of support as time  $t_1$ , and by  $t_1^-$  and  $t_1^+$  the times before and after the impact occurs, respectively. For smooth exchange-of-support, we require the angular velocities, before and after, to be identical, that is:

$$\dot{\mathbf{q}}_{2i}(t_1^-) = \dot{\mathbf{q}}_{1i}(t_1^+) \quad (1)$$

The kinematic relations have been used and the differential problem solved to obtain the cartesian velocities immediately before and after contact. These equations constrain the biped to zero velocity foot falls. The foregoing derivation determined conditions for smooth exchange-of-support.

In accordance, the equation of the tip of the swing leg along the  $x$ -axis is computed by summing a linear function with a sinusoidal function:

$$x_{21}(t) = 2V_F \left[ t - \frac{1}{2pf} \sin(2pf t) \right] \quad (2)$$

where  $f$  is the step frequency (number of steps per unit of time). Moreover, the vertical motion, that allows the foot to be lifted off the ground, is implemented using the function:

$$y_{21}(t) = \frac{F_C}{2} [1 - \cos(2pf t)] \quad (3)$$

The trajectory generator is responsible for producing a motion that synchronises and co-ordinates the leg behaviour. In this perspective, we assure that the swing limb arrives at the contact point when the upper body is physically centred with respect to the two lower limbs.

### 3.3. Workspace analysis

One pertinent question involving the motion planning is to determine which trajectories are physically realisable. Upon this point, we define the locomotion workspace of the biped robot as the set of all physically possible hip heights and step lengths, relative to a given reference trajectory (Fig. 3).

The locomotion workspace, as defined above, can be investigated by solving the kinematic equations given various restrictions. For the walking model, there are two kinds of kinematic restrictions:

- Sinusoidal restrictions due the type of velocity and acceleration trajectories used.
- Knee-foot restrictions since the legs are constrained by ground contact and the knee joints lock at certain positions (similar to human knee).

In this way, all the results derived ahead are capable of displaying the performance measure for interpretation over the entire stand-up workspace.

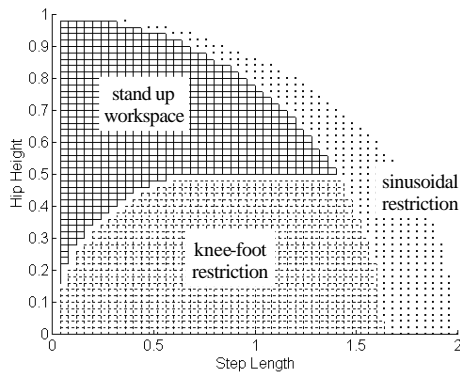


Fig. 3 - Workspace boundaries of the biped robot with  $l_1 = l_2 = 0.5$ .

## 4. Performance evaluation

This section covers the implementation of different performance measures used in the evaluation of the biped locomotion. The kinematic analysis is a challenging problem since the biped motion is characterised by gait patterns with complexity of large degrees of freedom (*i.e.*, multidimensional problem). In mathematical terms, we shall provide three global measures of the overall dexterity of the mechanism in some average sense. The aim is to verify whether a correlation between different viewpoints could be found.

### 4.1. Locomobility measure

The motivation that led to the development of the locomobility measure was to apply the concepts of kinematic dexterity to biped walking. In this section, we provide a geometric formulation of global dexterity based on the velocity manipulability ellipsoid of Yoshikawa [15]. In our case, we selected a scalar index that can be interpreted as a “directional gain” for different hip and foot trajectories. At each instant, we define a vector  $\vec{L}_V$  from the centre to the boundary of the velocity ellipsoid, having the same direction of the motion (Fig. 4). The global index is obtained by averaging the magnitude of this vector over a complete cycle:

$$L = \frac{1}{T} \int_0^T |\vec{L}_V(t)| dt \quad (4)$$

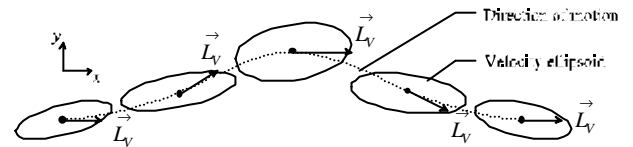


Fig. 4 - Evaluation of  $\vec{L}_V$  from the velocity ellipsoid and the direction of motion.

### 4.2. Perturbation analysis

In many practical cases the robotic system is noisy, that is, has internal disturbing forces. As such, an approach called “perturbation analysis” was implemented to determine how the biped model has enough flexibility to match these distortions. First, the joint trajectories are computed by the inverse kinematics algorithm. Afterwards, the angular acceleration vectors are ‘corrupted’ by additive noise. For simplicity reasons, it is used a uniform distribution with zero mean added to the acceleration signal. As result, the trajectories of the body and foot suffer some distortion and can only approximate the desired one. By regarding the forward kinematics of the mechanism, we determine a two-dimensional index based on the statistical average of the well-known mean-square-

error. We can express this index in terms of the following equations:

$$\mathbf{x}_x = \frac{1}{N_s} \sum_{i=1}^{N_s} \sqrt{\frac{1}{T} \int_0^T [\dot{x}_i^r(t) - \dot{x}_i^d(t)]^2 dt} \quad (5)$$

$$\mathbf{x}_y = \frac{1}{N_s} \sum_{i=1}^{N_s} \sqrt{\frac{1}{T} \int_0^T [\dot{y}_i^r(t) - \dot{y}_i^d(t)]^2 dt} \quad (6)$$

where  $N_s$  is the total number of steps for averaging purposes,  $\dot{x}_i^r$  and  $\dot{x}_i^d$  ( $\dot{y}_i^r$  and  $\dot{y}_i^d$ ) are the  $i$ th samples of the real and desired horizontal (vertical) velocities, respectively.

The perturbation analysis is a ‘second order’ method that measures the system’s robustness against variations around the desired trajectory. The stochastic perturbation penalises the system’s smoothness and we shall be concerned with minimising both  $\mathbf{x}_x$  and  $\mathbf{x}_y$ .

### 4.3. Lowpass frequency response

In robotics, the electro-mechanical system that regulates the movement (e.g., actuators and drives) is constrained by its bandwidth. Hence, the practical conditions of motor control should also be considered when evaluating the system’s performance. In this perspective, the joint variables are expanded in Fourier series and the time domain signals are described by the coefficients of its harmonic terms. Afterwards, these series

are filtered through the same lowpass filter. In doing so, the response of the system will differ from the input command, thereby producing an error. Once again, we can compare the real and the desired trajectories produced at the body using, as our figure of merit, the mean-square-error as expressed in equations (5) and (6) for  $N_s = 1$ . This method, based on frequency domain analysis, has also been successfully employed. However, it is a less general method, because it requires the *a priori* knowledge about the nature of the robotic actuators.

## 5. Simulation results

In this section, we perform a set of experiments to estimate how the biped robot adapts to variations in the locomotion variables. The aim is to determine the main factors that optimise the leg motion and to compare the formulated dexterity measures. To evaluate the system’s operating features, the simulations are carried out considering a forward velocity  $V_F = 1$  m/sec.

### 5.1. Step Length and Hip Height

To illustrate the use of the preceding results, the biped locomotion was simulated. As shown in Fig. 5, the performance functions are evaluated with respect to the step length and the hip height (for equal link lengths).

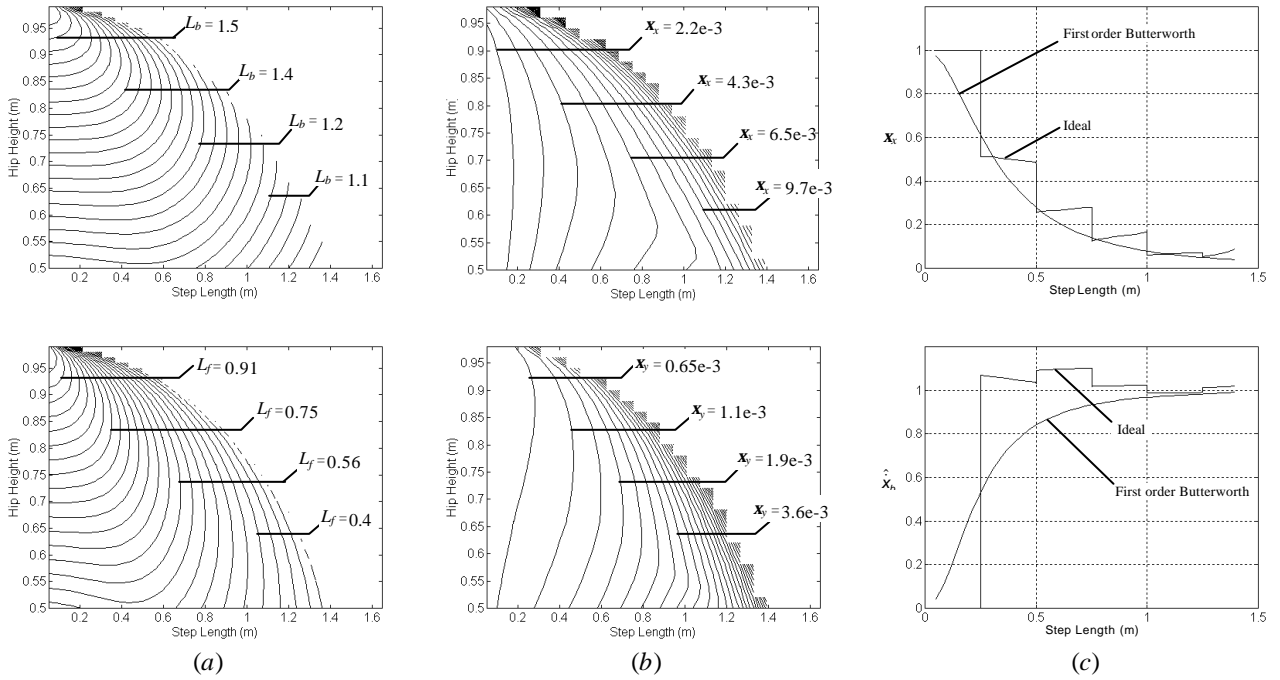


Fig. 5 - Contour plot: (a) locomobility index at the body ( $L_b$ ) and foot ( $L_f$ ); (b) perturbation analysis at the body along the x and y-axis ( $\mathbf{x}_x$  and  $\mathbf{x}_y$ ); (c) lowpass frequency response at the body ( $\mathbf{x}_x$ ) and mean forward velocity ( $\hat{x}_b$ ).

At this phase, a major simplification is introduced by allowing the swing foot to stay always on the ground. For convenience, the surface portion corresponding to hip heights lower than 0.5 m is not represented.

Fig. 5(a) corresponds to the locomobility measure at the body and foot. The main result is that, to maximise  $L$ , the hip height is expected to round about 95% of the maximum height. An important degradation occurs as the hip height decreases and/or the step length increases. In Fig 5(b), we depict the results when applying the perturbation analysis. In this case, we are plotting the performance measure at the hip ( $x$  and  $y$  coordinates). From these plots, we conclude that an important degradation occurs as the step length increases. The hip height variable also affects the measure, however, has a smaller influence. Finally, we examine the “lowpass frequency response” method. In Fig. 5(c) we have used both an ideal and a Butterworth lowpass filter with bandwidth  $f_b = 2$  Hz. Beyond the mean square error along the  $x$ -direction  $\mathbf{x}_x$ , we have also analysed the influence of the bandwidth on the mean forward velocity  $\hat{x}_b$ . In contrast to the other methods, the results suggest that the performance improves for higher step lengths. Both indices ( $\mathbf{x}_x$  and  $\hat{x}_b$ ) remain almost constant for different hip heights.

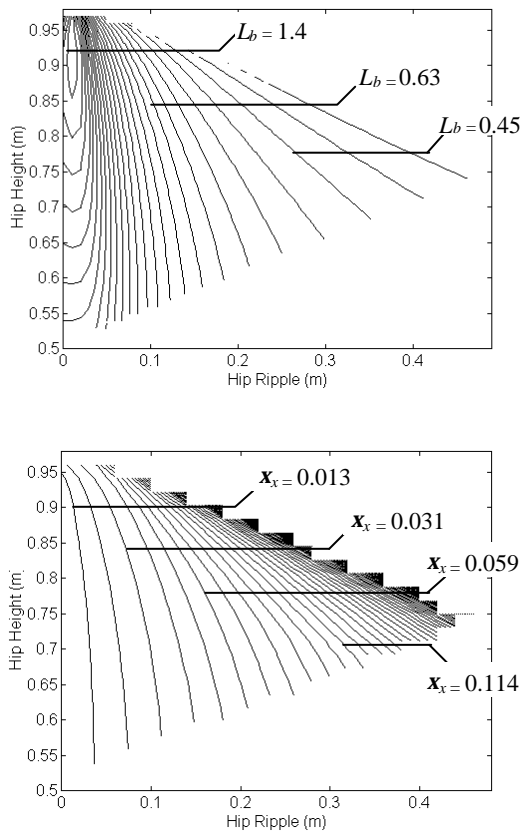


Fig. 6 - Contour plot:  $H_H$  versus  $H_R$  ( $L_S = 40$  cm).

## 5.2. Hip Ripple

Upon this point, we consider a hip trajectory with a sinusoidal oscillation, while the foot of the swing leg remains at the ground during all the cycle. The contour plots in Fig. 6 allows the confrontation between the two methods: locomobility and perturbation. The analysis suggests that a small adjustable oscillation at the hip may be advantageous. Furthermore, this value remains almost unchanged to hip height variations. On the other hand, the point of maximum smoothness will correspond to a zero hip ripple value.

## 5.3. Foot Clearance

Until now, all the experiences considered that the foot stays on the ground without any friction. Next, we analyse the situation in which the foot can be lifted off the ground. Fig. 7, shows the influence of the foot clearance variable for different hip height conditions. The results obtained were measured at the foot. In terms of locomobility index  $L_f$  and perturbation analysis  $\mathbf{x}_x$ , the optimum foot clearance tends to zero. In other words, the minimum foot clearance (except when avoiding any accidental contact) is the optimal one.

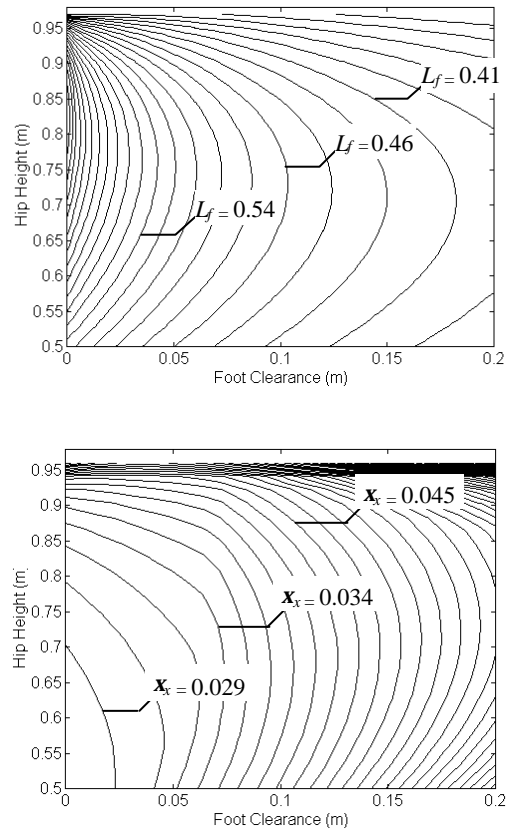


Fig. 7 - Contour plot:  $H_H$  versus  $F_C$  ( $L_S = 40$  cm).

## 5.4. Relative link lengths

We now investigate the role of the link lengths,  $l_1$  and  $l_2$ , in the system's performance. The choice of the leg lengths is a relevant aspect at the design process and affects the robot's mobility, as well. Fig. 8 depicts the locomobility index  $L_b$  versus the link length  $l_1$ . As can be observed, the optimum solution occurs when the knee-ankle length is slightly smaller ( $l_1 = 0.46$  cm) than the hip-knee length ( $l_2 = 0.54$  cm). The simulation also indicates that for  $l_1$  in the range from 30 to 60 cm the performance index remains almost constant.

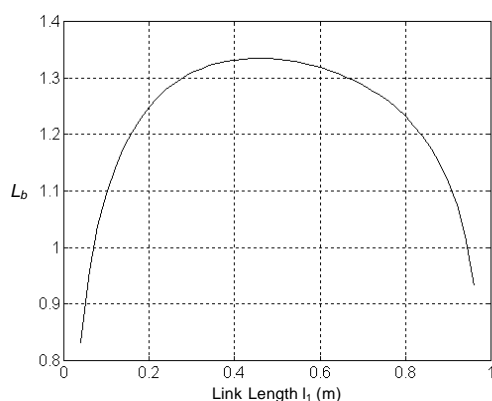


Fig. 8 - Performance curve versus link length  $l_1$ .

## 6. Conclusions

In this paper, we have compared various kinematic aspects of biped locomotion. By implementing different motion patterns, we estimated how the robot responds to a variety of locomotion variables such as hip ripple, hip offset, foot clearance and relative link lengths. Various quantitative measures were formulated for analysing the kinematic performance of such systems. Given these goals, the graphical results provide a more concrete illustration of the system's properties. The results obtained using the locomobility measure seems to agree well with experimental observations. On the other hand, the perturbation analysis method tends to be more elegant although computationally more exigent. The lowpass frequency response produces basically different effects allowing a different perspective of the problem.

While our focus has been on kinematic dexterity, certain aspects of locomotion are not necessarily captured by the measures proposed. Thus, future work in this area will address the refinement of our models to incorporate dynamics, as well as exploring human-like walking principles. In contrast to rotational actuators used in robotics, the skeletal is completely controlled by linear actuators organised in agonist-antagonist pairs. A

complementary analysis could reveal some important properties of both approaches giving some hints to understand the performance differences between robotic and biological motion.

## 7. References

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