FOURIER ANALYSIS OF MULTI-LEGGED PERIODIC GAITS

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Abstract

This paper analyses periodic gaits of multi-legged locomotion systems. The joint signals are studied in the Fourier domain, namely from the point of view of its reproducibility through low-pass actuators. The influence of several parameters is also considered and their critical values are investigated.

1 Introduction

Walking machines allow locomotion in terrain inaccessible to other type of vehicles because they do not need a continuous support surface but, on the other hand, they request active leg coordination for adaptive operation [1]. Gait selection and footfall planing are research areas, requiring an appreciable modeling effort for improvement of mobility with legs in unstructured terrain [2-3]. Previous studies of biological nature, mechanical construction, theoretical and computer simulation, focused in the structure and selection of locomotion modes according to different criteria such as energy efficiency, energy distribution along the cycle time, stability, velocity, comfort, mobility and environmental impact. Nevertheless, besides these aspects, it is also necessary to consider the capability of mechanical implementation due to the physical limitations of joint actuators [4-6]. Bearing this fact in mind, it was developed a kinematic and simulation model for multi-leg locomotion systems, considering several periodic gaits. The joints variables are analyzed in what concerns its variation with the duty factor, the leg stroke, the links length and the cycle time. The analysis consists in the signal decomposition through a Fourier series and the study of its convergence, namely the errors arising from its reproduction through low-pass actuators. Several simulation experiences reveal the system configuration and the type of the movements that lead to a better mechanical implementation of a given locomotion mode.

Having these ideas in mind, this paper is organized as follows. Section two introduces the fundamental concepts underlying the study and section three presents the package LEGS for simulating the walking movements. Based on this program a set of experiments is developed in section four, in order to establish the influence of the parameters in the periodic gaits, in a Fourier perspective. Finally, section five outlines the main conclusions.

2 Formulating a model for legged locomotion

It is considered a longitudinal walking system with $2n$ legs (n = 2,3,4…) corresponding to the quadruped, hexapod and octopod cases, because the biped system presents distinct characteristics [7].

The legs with two-*dof* rotational joints are numbered from the front to the rear of the system, according to 1,…, $2n-1$ (odd) in the left side and 2,…, $2n$ (even) in the right side, with a distance P between two consecutive legs in the same side of the body.

2.1 Kinematics modeling

Leg motions are described by mean of a coordinate system associated to each leg (Fig.1). Defining the leg stroke R, the cycle time T, the duty factor β, the transference phase $t_T = (1-\beta)T$ and the support phase $t_S = \beta T$, we consider a periodic trajectory for each foot, maintaining a constant body velocity $v_{Bo} = R/\beta T$.

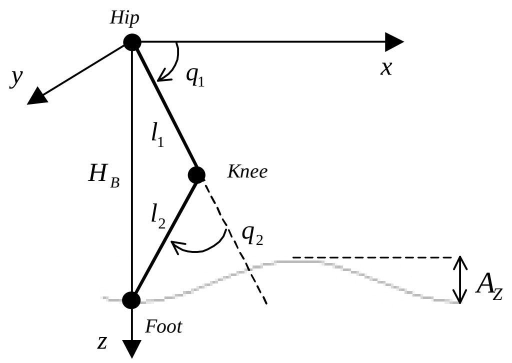


Fig. 1: Coordinate system an variables associated to each leg of the walking machine.

The impact and friction effects are avoided by keeping zero velocities of the feet in the instants of landing and taking off, assuring also the velocity continuity. For a step acceleration a during the transfer phase, the previous conditions are satisfied by the equation $a = 4(R + t_T \cdot v_{Bo})/t_T^2$ in the direction of body movement. In the vertical direction the above conditions are guaranteed by a sinusoidal trajectory with amplitude A_Z . For each trajectory in {x, z} we calculate the joint angles, selecting the solution corresponding to a forward knee in the inverse kinematics:

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} tan^{-1}(z/x) - tan^{-1}(l_2 C_2/(l_1 + l_2 C_2)) \\ cos^{-1}((x^2 + z^2 - l_1^2 - l_2^2)/2 l_1 l_2) \end{bmatrix}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \frac{1}{l_1 l_2 S_2} \begin{bmatrix} l_2 C_{12} & l_2 S_{12} \\ -l_1 C_1 - l_2 C_{12} & -l_1 S_1 - l_2 S_{12} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} \qquad (1)$$

where $C_i = cos(q_i)$ and $S_i = sen(q_i)$.

2.2 Movement Restrictions, Gait and Stability

The leg state "completely stretched" ($q_2 = 0$) corresponds to a singularity and must be avoided using values of leg stroke bellow its maximum $R_{máx} = 2\sqrt{(l_1 + l_2)^2 - H_B^2}$, with $H_B \le l_1 + l_2$, in order to allow an effective contact of the feet with the ground.

If the transversal insertion of the legs is the same, along the longitudinal axis of the body, collisions may occur between consecutive legs in the same side of the body. These collisions can be avoided imposing the condition $P > 2(l_1+l_2)$, but it leads to a body length $L_b = (n-1)P$ that can be disadvantageous for walking machines with a large number of legs. Alternatively, the condition $R_{max}/P \le 1$ assures the absence of collisions between consecutive legs. However, it limits the leg stroke R, that should be as larger as possible in order to allow considerable adjustments in rough terrain. In this case the trajectory of the foot must be considered according to the selected leg stroke R and foot offset A_Z, for the purpose of collision avoidance.

Gaits describe discontinuous sequences of collective leg movements, alternating between transfer and support phases. In this study we consider periodic gaits both forward and backward periodic gaits were implemented paying special attention to the wave gait since it presents optimal static stability among periodic gaits. Wave gait phases (Fig. 2) are described by $\phi_{2m+1}=F(m\beta)$ for odd legs and $\phi_{2m}=F(m\beta+1/2)$ for even legs, where $F(X)$ represents the fractional part of real number X and $m = 1, 2,\ldots, n-1$ is the index of each leg.

Static stability is closely related to the walking velocity. Maintaining the leg stroke R, a larger walking velocity v_{Bo} implies higher leg velocities and a smaller cycle time T. When the leg velocities reaches its maximum values, imposed by physical limitations of the actuators, an increase of v_{Bo} can only be achieved through the reduction of β, with the consequent diminishing of the stability. Moreover, increasing the number of legs of the walking machine, leads to superior static stability, allowing smaller values of β and larger values of v_{Bo}.

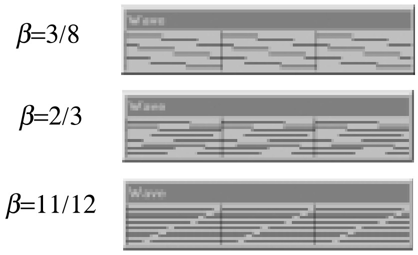


Fig. 2: Octopod wave gait for several duty factors β.

3 Overview of LEGS program

Practical factors such as the efficiency of the mechanical components and actuators, the complexity of system integration and the cost have limited the extension of experimental research in walking machines. On the other hand, simulation models using computer programs, provide a more flexible analysis tool.

In this perspective we developed the program LEGS, that implements the previous models for simulation and analysis (Fig. 3). The package is prepared for the study of $2n$ legs (n = 2,3,4,...) locomotion systems. Therefore, by varying the number of legs, users are able to 'switch' between quadruped, hexapod, octopod, or other locomotion systems with any even number of legs. The configuration of the locomotion system is completed with the definition of the stroke pitch P, the body height H_B and the leg link lengths l_1, l_2.

The gait and the associated duty factor define the locomotion mode. The periodic gaits implemented in the package are the wave gait, half/full cycle equal phase gait, backward wave gait, half/full cycle backward equal phase gait and the continuous/ discontinuous follow–the–leader gaits. The definition of the associated duty factor is limited according with the selected gait and the gait diagrams are shown during locomotion, for the selected cycle time T. Moreover, the leg stroke R and the vertical foot offset A_z define the trajectory for each leg, and complete the set of parameters for the system simulation.

The program calculates the foot and the joint positions and velocities for each leg, and provides the real time animation and coordination of the walking system. Stability analysis is shown through the support pattern and the vertical projection of the body center of gravity.

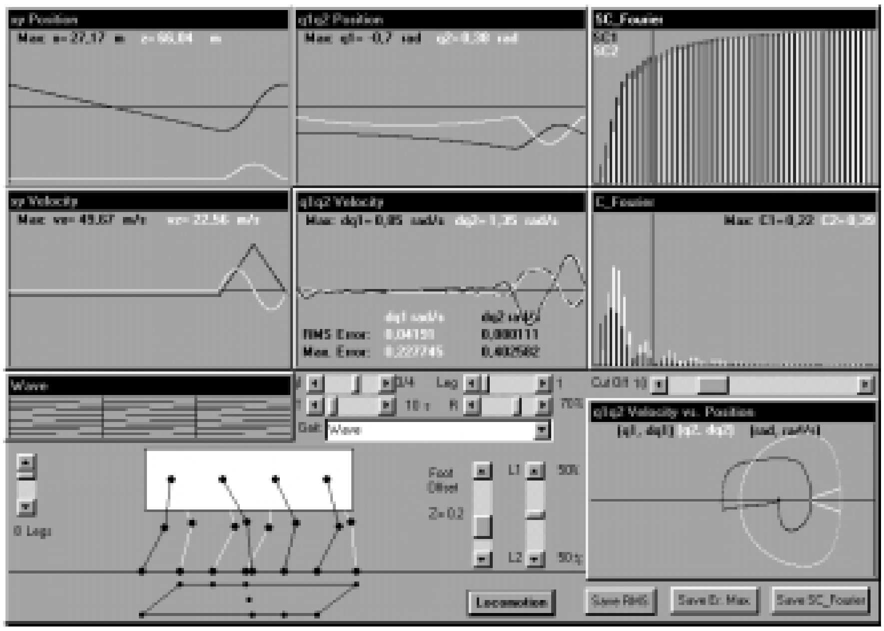


Fig. 3: Main screen of the computer program LEGS. Configuration of the octopod longitudinal system and the correspondent support pattern for $t = T$, of the wave gait with $\beta = 3/4$, $R = 0.70R_{max}$ and $l_1 = l_2$.

The phase diagram of joint velocities is also presented for the analysis of the resulting trajectories. The program calculates the Fourier series of the joint velocities that is presented in a graphical form, and saves the data for analysis with external tools. The Fourier series is also used to simulate the effect of low-pass joint actuators. After defining the cut-off harmonic order, joint velocities are reproduced using the harmonics below cut-off. Correlation between planned and reproduced joint velocities is estimated using the RMS and maximum errors.

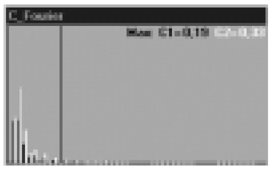
4 Fourier analysis of walking gaits

This section investigates the influence of several parameters upon the joint variables that must be supported by the leg actuators during periodic gaits.

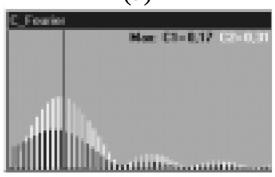
4.1 Duty factor

In the paper we focus on the octopod system because it reveals a wide range of possible duty factor for the wave gait ($3/8 \leq \beta \leq 1$) when comparing with the hexapod ($1/2 \leq \beta \leq 1$) and the quadruped ($3/4 \leq \beta \leq 1$) systems. Moreover, for the same duty factor, it is more statically stable than the other systems. Nevertheless, the study does not lose generality because, for the wave gait, the leg movements of the octopod and the quadruped/hexapod systems are similar.

The decomposition of the joint velocities in Fourier series for $\beta \leq 3/4$ reveals that the major part of the signal energy is located in the lower frequencies (Fig. 4.*a*), while for higher duty factors a considerable part of the signal energy appears in high harmonics (Fig. 4*b*). In fact, we truncated the Fourier series for orders lower than the cut-off harmonic ($n_k = 10$ in the experiment) in order to reproduce the joint velocities and to compare them with the planed variables (Fig. 5).

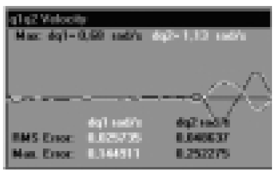


(a)

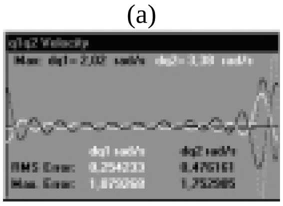


(b)

Fig. 4: Coefficients C^i_k of the Fourier decomposition for the joint velocities; (a) $\beta = 1/2$; (b) $\beta = 11/12$.



(a)



(b)

Fig. 5: Correlation between the planed and reproduced velocities in the joints, resulting from the truncated Fourier series for $n_k \leq 10$. (a) $\beta = 3/4$; (b) $\beta = 11/12$.

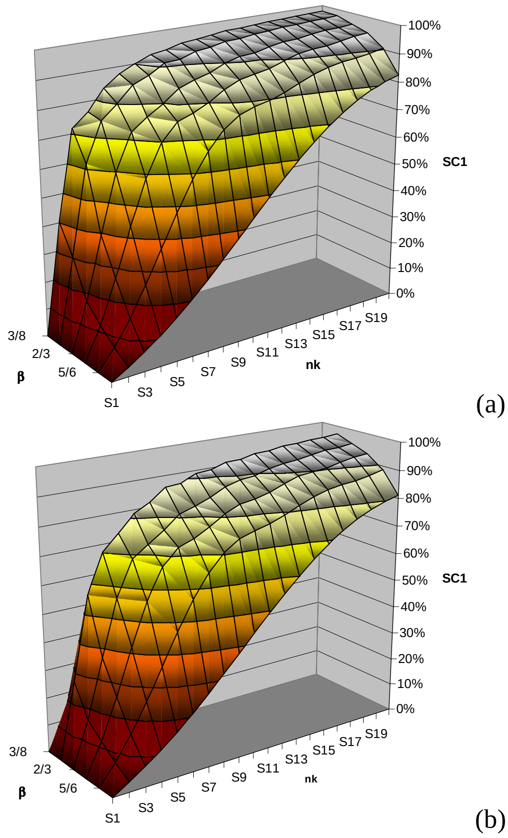


Fig. 6: Partial sums $SC_1(\beta, n_k)$ of the Fourier series for the joint 1 velocity of an octopod with $l_1 = l_2$, for the wave gait. (a) $R = 0.50R_{max}$; (b) $R = 0.90R_{max}$

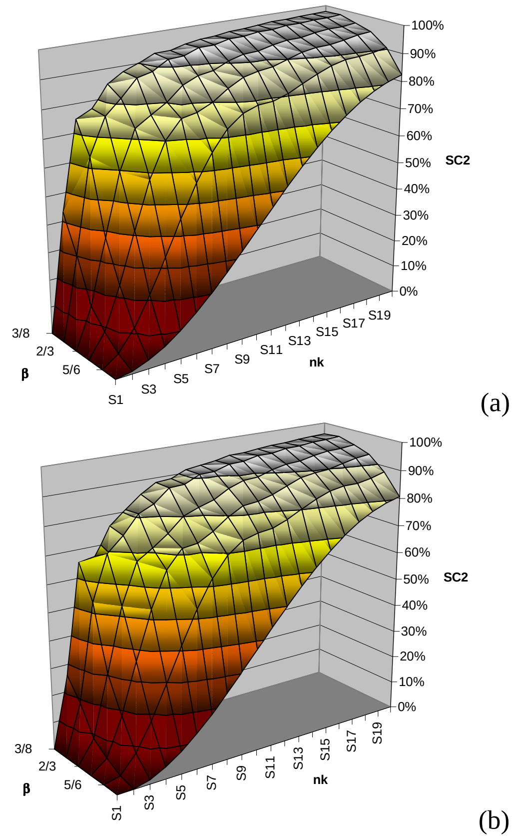


Fig. 7: Partial sums $SC_2(\beta, n_k)$ of the Fourier series for the joint 2 velocity of an octopod with $l_1 = l_2$ for the wave gait. (a) $R = 0.01R_{max}$; (b) $R = 0.70R_{max}$

To study the effect of β in the joint velocities, for each leg stroke R we calculate the partial sums of the Fourier coefficients $SC_i(n_k)$.

For joint 1 and leg stroke values $R \leq 0.50R_{max}$ there are minor differences in the convergence rate of SC_1 (Fig. 6*a*). For this range of R, good levels of convergence are obtained for $\beta = 3/8$ and $\beta = 1/2$. Furthermore, for values of the leg stroke $R > 0.50R_{max}$ there is a maximum of SC_1 around $\beta = 1/2$ that is emphasized as R approaches it's maximum value R_{max} (Fig. 6*b*), suggesting the existence of an optimal point for the implementation of the joint 1 velocity.

The experience for joint 2 and $R \in [0.01R_{max}, 0.70R_{max}]$ reveal a slight decrease in the partial sums SC_2 (Fig. 7), resulting a kind of 'unstable' behavior in terms of β for leg strokes $R \geq 0.80R_{max}$. This 'unstable' behavior (Fig. 8) with low level of convergence rate, is associated with the singularity $q_2 = 0$, caused by the adoption of leg strokes near the maximum R_{max}..

For $R > 0.70R_{max}$ there are two relative maximum values of SC_2 around $\beta = 1/2$ and $\beta = 5/6$ that suggest the use of these duty factors as a form of speeding up the convergence of the Fourier series and making easier its mechanical implementation.

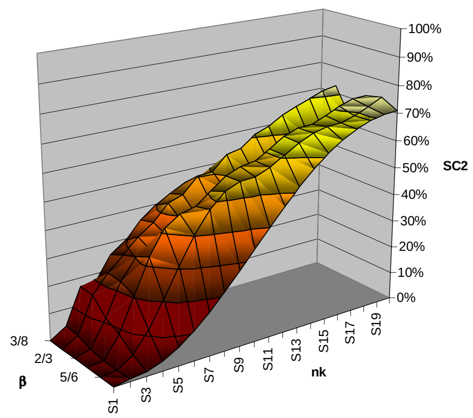


Fig. 8: Partial sums $SC_2(\beta, n_k)$of the Fourier series for the joint 2 velocity of an octopod with $l_1 = l_2$ and $R = 0.99R_{max}$ for the wave gait.

4.2 Leg stroke

To study the influence of leg stroke R we carried out a similar study to the one developed previously.

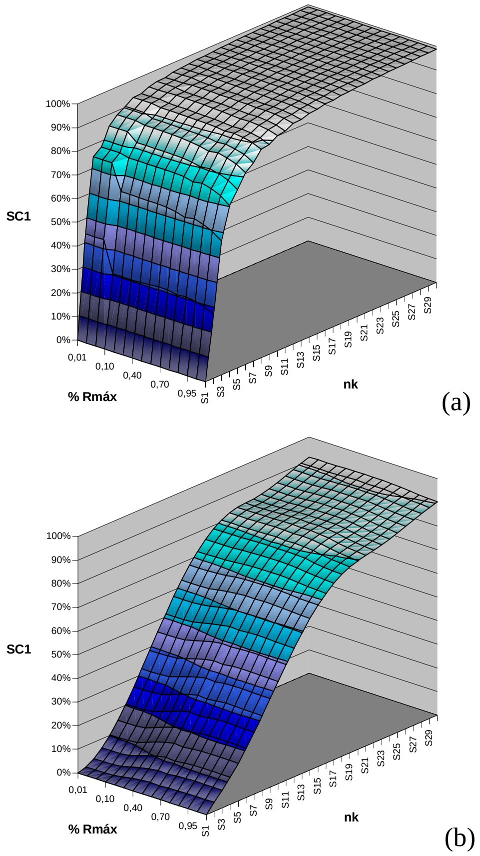


Fig. 9: Partial sums SC_1 of Fourier coefficients associated to velocity in joint 1 of an octopod system with $l_1 = l_2$ and leg stroke R, using the wave gait with duty factor β.
(*a*) $\beta = 3/8$; (*b*) $\beta = 11/12$

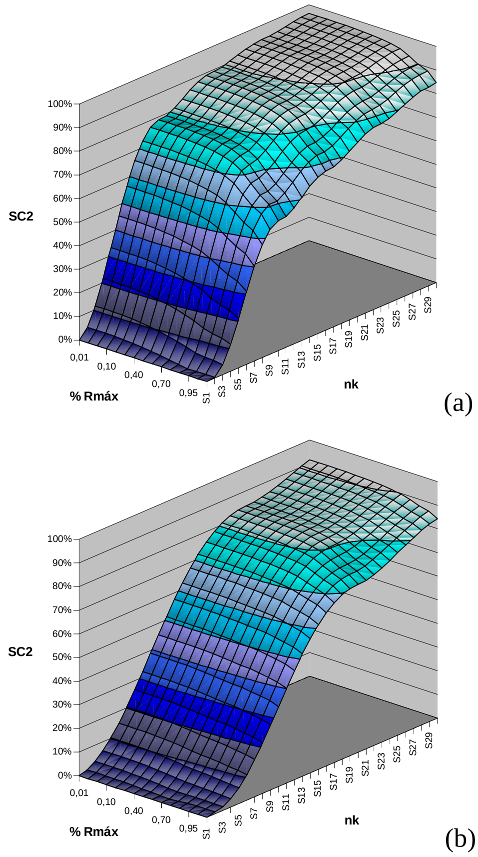


Fig. 10: Partial sums SC_2 of Fourier coefficients associated to velocity in joint 2 of an octopod system with $l_1 = l_2$ and leg stroke R, using the wave gait with duty factor β.
(*a*) $\beta = 5/6$; (*b*) $\beta = 11/12$

For all considered values of β, the partial sums $SC_1(R,n_k)$ exhibit a slight decrease as the leg stroke increases (Fig.9), showing a weak dependence of $\dot{q}_1(t)$ with R, associated with the absence of a singularity in joint 1.

The experiences in joint 2 reveal an appreciable decrease of $SC_2(R,n_k)$ as R increases (Fig.10.*a*), for all values of β, with the exception of $\beta = 11/12$ (Fig.10.*b*) that shows a poor global rate of convergence. This strong dependence of $\dot{q}_2(t)$ with R is explained by the singularity $q_2 = 0$ when $R \rightarrow R_{max}$.

For $\beta \leq 5/6$ the partial sum $SC_2(R,n_k)$ reveals a strong decrease when $R \geq 0.90R_{max}$. The conclusions point out the adoption of $R < 0.90R_{max}$ to get a better joint implementation of the selected locomotion mode. The limitations introduced by this restriction do not lead to an appreciable decrease of the body velocity, when maintaining the other parameters.
Another conclusion stewing from these experiences is the importance of the actuators selection in joint 2, because of the strong dependence of $\dot{q}_2(t)$ with R.

4.3 Link lengths

To reduce the possible combinations of the movement parameters $\{\beta, R\}$ with the parameters of system structure $\{l_1, l_2\}$ we selected intermediate values for the duty factor $\beta = 3/4$ and the leg stroke $R = 0.50R_{max}$ varying l_1 and l_2 while maintaining l_1+l_2 constant. To generalize this study we extended the experiences to other values of $\{\beta, R\}$ varying separately each of these parameters.

For the intermediate values of $\{\beta, R\}$ (Fig. 11*a*) the experiences for joint 1 reveal an appreciable decrease in the rate of convergence of the Fourier series SC_1 for $l_1 \geq 0{,}80(l_1+l_2)$, while for intermediate values of R and small β (Fig. 11*b*), the fastest rate of convergence is achieved, and this decrease is observed for $l_1 \geq 0{,}90(l_1+l_2)$, suggesting the avoidance of this values of $\{l_1,l_2\}$ and the use of $l_1 < 0{,}80(l_1+l_2)$.

For an intermediate R and an high β (Fig. 11*c*) there is a slow decrease for the rate of convergence of SC_1 as $l_1/(l_1+l_2)$ increases. For intermediate β and extreme R (Fig. 11*d*) the rate of convergence of SC_1 reveals an unstable dependence about $l_1/(l_1+l_2)$.

For $\beta \le 3/4$ and $R \le 0.50R_{max}$ SC_1 exhibits high level of convergence with a relative maximum for $l_1 = 0{,}78(l_1+l_2)$ that suggest the use of this values of $\{l_1,l_2\}$ for moderated values of $\{\beta, R\}$. For extreme values of β or R partial sums SC_1 reveals less pronounced local maximum and weaker dependence about $l_1/(l_1+l_2)$ with low levels of convergence for all values of (l_1,l_2) associated to extreme values of duty factor or leg stroke.

The results obtained for joint 2 agree with those obtained for joint 1, leading to the same choices of the pair $\{l_1 = 0{,}78l, l_2 = 0{,}22l\}$ to optimize the implementation of velocity for both joints.

Finally, the cycle time T was tested against the Fourier series amplitudes showing, as expected, a $1/T$ dependence.

5 Conclusions

This paper presented the fundamental aspects of multi-legged walking systems and the package LEGS for its computer simulation. Based on this tool, the influence of several parameters upon the joint variables was analyzed, when performing periodic gaits. The Fourier analysis revealed the most relevant parameters and their critical range of variation from a cinematic perspective. Future developments will include the dynamics and non-periodic walking cycles.

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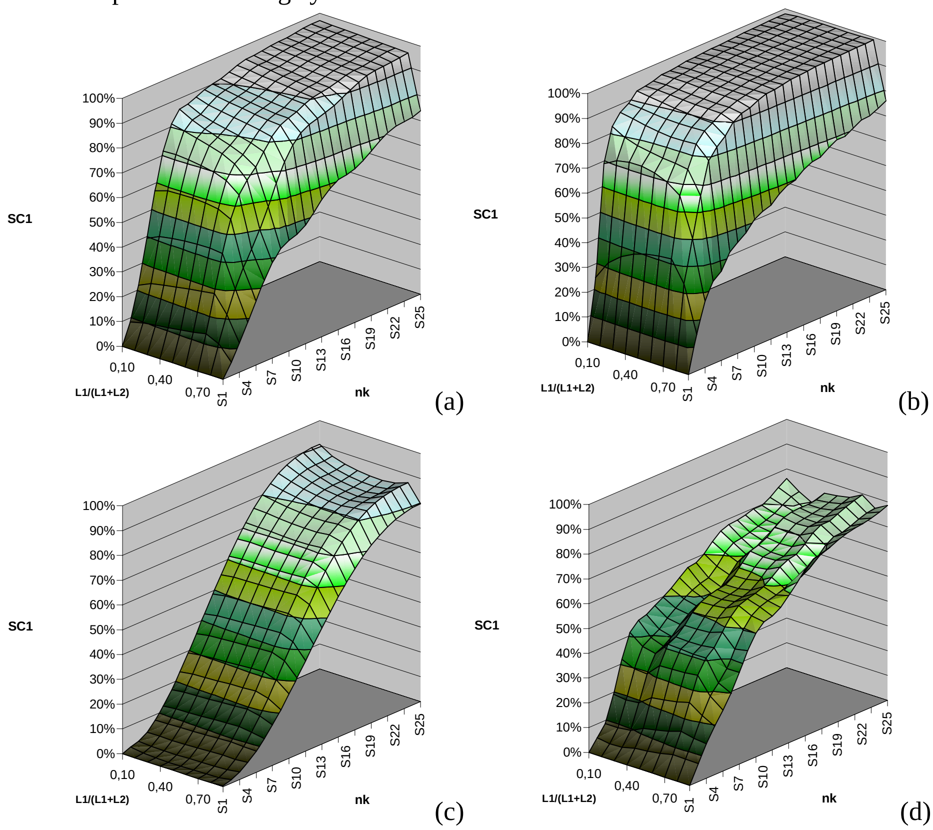

Fig. 11: Partial sums SC_1 of Fourier coefficients associated to velocity in joint 1 of an octopod system as a function of $l_1/(l_1+l_2)$ for given values of leg stroke R and duty factor β, using the wave gait.
(a) $R = 0.50R_{max}$ $\beta = 3/4$; (b) $R = 0.50R_{max}$ $\beta = 3/8$; (c) $R = 0.50R_{max}$ $\beta = 11/12$; (d) $R = 0.99R_{max}$ $\beta = 3/4$