Optimal motion planning for the rendezvous of nonholonomic vehicles under disturbances

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Abstract—This paper presents a strategy for the coordination of teams of nonholonomic vehicles on phased operations. The vehicles should arrive simultaneously to their goal locations. The arrival time is minimized by the control design. The control design is made taking into account the existence of constant disturbances.

I. INTRODUCTION

In certain multi-vehicle operations, it is important that the vehicles reach a set of locations approximately at the same time: the so called rendezvous problem. The operations can also be executed in multiple phases, resulting in a succession of rendezvous. This paper proposes an approach for the rendezvous problem based on the automatic generation of trajectories for vehicles operating in a two dimensional space. The proposed control system should be capable of generating trajectories in order to ensure that all vehicles reach their respective assigned location at the same time, even under the effect of disturbances. The trajectories are chosen in order to minimize the rendezvous time and, where possible, power usage. In this work, the analytical expressions of the trajectories are derived assuming a constant disturbance.

The considered vehicles are nonholonomic and they are described by a kinematic model which closely resembles the typical Dubins vehicle [1]. This work is based on this type of vehicles because to a certain extent they can mimic the behaviour of more complex ones, such as underwater and aerial vehicles operating in a plane, and provide insight about their operation.

The focus on the nonholonomic vehicles is motivated by our developments on under-actuated autonomous underwater vehicles on the Piscis project from Porto University and their applications [2]. The objective of this paper is to devise efficient trajectories for the type of operations described in that work, namely sampling operations.

The problem of motion planning for an AUV under spatially varying additive disturbances was studied in [3]. However, the model considered by those authors allows trajectories with infinite curvature (turn in place). The optimal motion planning for the Dubins vehicles (vehicles moving only forward and with curvature constraints) was studied in [1] and more recently in [4] and [5]. The main contribution of the presented formulation, besides the application to the rendezvous problem, is the modeling and handling of a constant disturbance by the control design. The consideration of the disturbance, even if constant, adds a drift term to the model which prompts for a new analysis of the problem.

The paper is organized as follows. Section describes the mathematical model that will be employed throughout the remaining paper and shows how a more complex model is mapped to that one. Section presents the constraints that should be met in order to achieve what we define by efficient trajectories. In section we derive the analytical expressions for the optimal trajectories considering constant disturbances. Finally, section presents the conclusions.

II. VEHICLE MODEL

Consider a vehicle moving, with forward velocity $v$ and orientation $\psi$, relative to a moving two dimensional “disturbance” frame. The disturbance frame moves with velocity $v_d$ and orientation $\theta_d$. The vehicle motion is described by the following kinematic model, where $x$ and $y$ are the position coordinates in the two dimensional operation space:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi}
\end{bmatrix} = v \begin{bmatrix}
\cos(\psi) \\
\sin(\psi)
\end{bmatrix} + v_d \begin{bmatrix}
\cos(\theta_d) \\
\sin(\theta_d)
\end{bmatrix}
\]

with

\[
\begin{align}
-c_{\text{max}} \leq c & \leq c_{\text{max}} \\
0 & < v_{\text{min}} \leq v \leq v_{\text{max}}
\end{align}
\]

The forward velocity $v$ can not be less than the “stall” velocity $v_{\text{min}} > 0$. This can be seen as the stall velocity for a plane or the required velocity to keep a slightly buoyant submarine at constant depth.

The minimum radius of curvature for these vehicles is obtained by applying the maximum curvature actuation $c$ and is trivially given by $r_{\text{min}} = 1/c_{\text{max}}$.

For compactness, $(v_{dx}, v_{dy}) = (v_d \cos(\theta_d), v_d \sin(\theta_d))$ and $x = [x \ y \ \psi]^T$ will also be used in some equations. The term $(v_{dx}, v_{dy})$ can be seen as a simplified modelling of slow-varying disturbances, such as winds for airplanes or currents for marine vehicles.
A. Underwater Vehicle Model

This section discusses the mapping of a nonlinear model of underwater vehicles to the kinematic model described in the previous section. Autonomous underwater vehicles (AUV’s) are best described as nonlinear systems (see [6] for details). In order to define the model, two coordinate frames are considered: body-fixed and earth-fixed. In what follows, the notation from the Society of Naval Architects and Marine Engineers (SNAME) [7] is used. The motions in the body-fixed frame are described by 6 velocity components \(v = (v_1, v_2) = [u, \dot{v}, w, p, q, r]\) respectively, surge, sway, heave, roll, pitch, and yaw, relative to a constant velocity coordinate frame moving with the ocean current. The six components of position and attitude in the earth-fixed frame are \(\eta = (\eta_1, \eta_2) = [x, y, z, \phi, \theta, \psi]\). The earth-fixed reference frame can be considered inertial for the AUV.

The velocities in both reference frames are related through the Euler angle transformation

\[
\dot{\eta} = J(\eta_2)v
\]  

In the body-fixed frame the nonlinear equations of motion are:

\[
M\ddot{v} + C(v)v + D(v)v + g(\eta) = \tau
\]  

\[
\eta = J(\eta_2)v
\]

- **M** Inertia and added mass matrix of the vehicle
- **C(v)** Coriolis and centripetal matrix
- **D(v)** Damping matrix
- **g(\eta_2)** Restoring forces and moments
- **\(\tau\)** Body-fixed forces from the actuators

Figure 1(a) depicts one of these vehicles. This AUV is not fully actuated. There is a propeller for actuation in the longitudinal direction (surge, in the naval terminology) and fins for lateral and vertical actuation. The effect of the fins depends on the longitudinal velocity of the vehicle (for zero speed they do not provide actuation).

The mechanical configuration of the AUV leads to a simpler dynamic model. \(\tau\) depends only on 3 parameters: propeller velocity \(n\) \((0 < n \leq n_{\text{max}})\), horizontal fin inclination \(\delta_r\) \((-\delta_{r\text{max}} \leq \delta_r \leq \delta_{r\text{max}})\) and vertical fin inclination \(\delta_v\) \((-\delta_{v\text{max}} \leq \delta_v \leq \delta_{v\text{max}})\). The dynamics of the thruster motor and fin servos are generally much faster than the remaining dynamics therefore, for the purposes of this work, they can be excluded from the model.

System identification for autonomous underwater vehicles is quite difficult and expensive for two reasons: the large number of model parameters (matrix coefficients) and the complexity of the experimental setup for system’s identification. In our developments we use a set of coefficients based on the results from [8] and on our field experiments.

This work focuses on operations on the horizontal plane. This restricts the motions of the AUV to planar motions at constant depth. We assume the existence of controllers that stabilize vehicle’s depth and pitch, i.e., \(w\) converges to a small value (which in practice is not equal to zero due to the required pitch to compensate vehicle’s buoyancy) and \(q\) converges to 0. The roll rate \(p\) converges to 0 due to the restoring moment of the vehicle and the roll angle \(\phi\) converges to a value which depends on the thruster speed. In general, the pitch and roll angles can be made very small by physical configuration. Based on these assumptions and the physical shape of the vehicle, the approximated nonlinear model becomes [6]:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
ucos(\psi) - v sin(\psi) & 0 & 0 \\
usin(\psi) + v cos(\psi) & 0 & 0 \\
 & & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\psi
\end{bmatrix}
\]

\[
\begin{bmatrix}
f_u(v, \tau) \\
f_v(v, \tau) \\
f_r(v, \tau)
\end{bmatrix} =
\begin{bmatrix}
(m - X_u)\dot{u} + X_{uv}v + X_{vu}w + X_{rr}r^2 + X_{ru}(u) \\
(m - Y_v)\dot{v} + Y_{uv}v + Y_{vu}w + Y_{rr}r^2 + Y_{ru}(u) \\
(m - N_z)\dot{z} + N_{uv}v + N_{vu}w + N_{rr}r^2 + N_{ru}(u) + N_{ru}(u)
\end{bmatrix}
\]

\[
\begin{bmatrix}
f_u(v, \tau) \\
f_v(v, \tau) \\
f_r(v, \tau)
\end{bmatrix} =
\begin{bmatrix}
(m - X_u)\dot{u} + X_{uv}v + X_{vu}w + X_{rr}r^2 + X_{ru}(u) \\
(m - Y_v)\dot{v} + Y_{uv}v + Y_{vu}w + Y_{rr}r^2 + Y_{ru}(u) \\
(m - N_z)\dot{z} + N_{uv}v + N_{vu}w + N_{rr}r^2 + N_{ru}(u) + N_{ru}(u)
\end{bmatrix}
\]

It can be seen that the radius of curvature is practically independent of the surge velocity for a certain range of velocities \([0.8, 2.0] [m/s]\). The curvature is mainly determined by the angular displacement of the rudder fin (which would be modelled by \(c\) in the kinematic model). Additionally, a slow varying water current with velocity \(V_c < V_{\text{max}}\) and direction \(\theta_c\) can be considered as an additive disturbance on the vehicle velocity: the basic motion of the vehicle will be made with relation to the moving column of water, as stated in the beginning of the section.

For these reasons, the kinematic model presented in equation (1) can be considered an acceptable approximation for trajectory planning purposes. Marine and aerial vehicles do
not posses the sideslip constraint, i.e., they move sideways (sway velocity on the AUV model). However, this motion is encompassed by the considered radius of curvature. If operation at constant speed is considered, the main difference is the fact that angular speed is allowed to vary instantaneously on the kinematic model while that is not possible on the physical system (and neither on the nonlinear model). The unions between segments and arcs would not be perfectly tracked by a real vehicle. The modelling error can be minimized by considering a larger radius of curvature but the imperfection will still be noticeable for small angular displacements. However, it is argued whether most tracking errors would be due to this simplification or to sensor noise and other practical phenomena. Moreover, the main objective is that the vehicles reach the destination at the desired time. If some slack is allowed when planning (for instance, considering $v_{\max} = v_{\max} - \delta$), that can be achieved.

In the remainder of the paper we use $v$ for the longitudinal velocity (replacing $u$ in the SNAME notation) and $\omega$ for the angular velocity (assuming the planar motion, this replaces $r$ in the SNAME notation).

III. WAYPOINT GENERATION

This section presents some considerations that must be taken in account when generating waypoints to be visited by the vehicles. A waypoint $w_i$ is a location $(x, y)$ that must be reached by vehicle $i$. Notice that the actual waypoint generation is not dealt in this paper. The waypoints can be generated either on-line or off-line. For a description of a possible global system architecture, along with the mechanisms for data exchange, coordination between vehicles and fault handling, see [9].

The main concern of this section is that the generation of waypoints should lead to efficient trajectories. The concept of efficient trajectory is linked with the vehicles’ motion constraints which, as said above, can be approximated by equation 1. Informally, it is desired that the ratio $\kappa$ between the Euclidean distance between consecutive waypoints and the actual travelled distance should be close to one.

Assuming no disturbances, consider the following waypoint sequence where $(x_0, y_0, \psi_0)$ is the vehicle initial posture: $(x_0 + 2kr_{\text{min}} \cos(\psi_0 \pm \pi/2), y_0 + 2kr_{\text{min}} \sin(\psi_0 \pm \pi/2))$, with $k \in \{1, 2, \ldots\}$. Basically, this corresponds to a sequence of waypoints separated by a distance of $2r_{\text{min}}$ along a straight line. The tracking of this waypoint sequence by a Dubins vehicle, using a general tracking law, generates a trajectory with the shape of a semi-circle for each step $k$, with a resulting $\kappa = \frac{2r_{\text{min}}}{r_{\text{min}}} = 64\%$. This ratio is acceptable for a single step (the vehicle performs far worse when travelling to a waypoint "behind" him) but it is not acceptable recurrently (which is quite possible in typical operations). It must be remarked that the future target waypoint is not known before the vehicle reaches the current target waypoint. Thus, there is no criteria to choose a final orientation. This is the reason why waypoints are given only as a pair $(x, y)$. If the sequence of waypoints was known in advance the vehicle could perform maneuvers in order to attain a suitable orientation. The above described behavior can be avoided if the distance between two consecutive waypoints is made greater than $2r_{\text{min}}$. In that situation, the vehicle’s orientation converges to trajectory’s orientation.

In order to gain insight about how greater should $d_{\text{min}}$ be than $2r_{\text{min}}$, consider the following scenario. Define $\xi_{\text{sr}}(x_0, l_{\text{sr}}, t) = \{x_0 + l_{\text{sr}}t\}$, $t > 0$, $\xi_{\text{sr}}(.) \in \mathbb{R}^2$, as the line segment starting at the initial point $x_0 = (x_0, y_0)$ of the vehicle and with direction given by a vector $l_{\text{sr}}$. Assume the vehicle is required to reach $\xi_{\text{sr}}(x_0, l_{\text{sr}})$ with the same orientation of $l_{\text{sr}}$.

Without loss of generality, consider the problem of a Dubins vehicle reaching the target set $\{(x, y, \psi) : x > 0, y = 0, \psi = 0\}$, in minimum time, departing from $(0, 0, \psi_0)$ and considering $v_d = 0$. Define $(x_f, 0, 0)$ as the point at which the vehicle reaches the target set. Notice that the value of $x_f$ will depend on the initial orientation $\psi_0$ of the vehicle. The maximum value of $x_f$ will be, in this scenario, the lower bound for the distance between waypoints. The time optimal paths will be composed of two arcs of radius $r_{\text{min}}$ (except if $\psi_0 = 0$ which gives the trivial solution $x_f = 0$). The first arc is tangent to the initial posture of the vehicle. The second arc is tangent to its final posture. Since the two arcs must be tangent at the intersection, the objective is to seek the initial direction that pushes the final arc more to the left. The locus of the maximum translational displacement done along the initial arc for all possible initial orientations is given by a circle of radius $2r_{\text{min}}$ centered at the vehicle’s initial position (see Figure 2). The maximum $x_f$ will be obtained when the final arc is tangent to this circle:

$$
\sqrt{(2r)^2 - x_i^2} = -\sqrt{r^2 - (x_i - x_f)^2} + r 
$$

with $r = r_{\text{min}}$. Equation 8 gives $x_i = \frac{3}{2}x_f$. Finally, plugging the previous result on equation 8 results in $x_f = \sqrt{8}r_{\text{min}}$. Thus

$$
d_{\text{min}} \geq \sqrt{8}r_{\text{min}}
$$

When $v_d > 0$, we must consider also consider the disturbance direction $\theta_d$. For each $v_d$ there will be a worst case $\theta_d$. 

![Fig. 2. Worst case for the minimum distance path to a target set $\{(x, y, \psi) : x > 0, y = 0, \psi = 0\}$](image-url)
i.e., a $\theta_d$ that maximizes $x_f$. Figure 3 presents the optimal trajectories for different values of $\theta_d$ when $v = 1(m/s)$, $v_d = 0.2(m/s)$. The value of $d_{\text{min}}$ is given by the maximum value of $x_f$.

IV. TRAJECTORY GENERATION

The approach for performing the rendezvous is divided in two steps: first, given the vehicle’s locations and target waypoints the minimum time $T$ to arrive to the target waypoints is calculated; finally, an optimal trajectory, in the sense of power consumption, is calculated for each vehicle such that each vehicle reaches its target waypoint in time $T$. The next sections describe the approach.

A. Time to rendezvous computation

In order to speed up operation we are interested in performing the rendezvous in the smallest possible time. The rendezvous will consist in the simultaneous arrival of each vehicle to its assigned waypoint. The vehicles can be preassigned to waypoints or not. If the assignment is made a priori the problem amounts to the computation of the solution of a minimum time problem whose dimension grows linearly with the number of vehicles. If there is no assignment we can optimize it in order to minimize the time to rendezvous, though at costs of higher computational complexity. The latter case could be solved in an elegant way by superposition of the forward reach sets of each vehicle (which could be computed off-line and stored on the computational system). However, we do not develop that scenario on this paper. In what follows, we assume the former case.

If the vehicles’ velocities are allowed to decrease to very small values (ideally to zero) the multi-vehicle minimum time problem can be decoupled in minimum time sub-problems for each vehicle. This way, the computational complexity decreases since it is sufficient to calculate the minimum time for each pair vehicle-waypoint. The limiting factor – the maximum travelling time $T$ for the vehicle-waypoint allocation – is found and it is guaranteed that all vehicles can reach their respective target waypoints at least in time $T$.

The above simplification regarding the lower bound for vehicle’s velocity can be considered very strong. However, it can be seen that if the distances to be travelled by each vehicle are not too disparate (an approximate bound can be given by $\frac{d_{\text{max}}}{v_{\text{min}}} < \frac{d_{\text{min}}}{v_{\text{max}}}$, where $d_{\text{max}}$ and $d_{\text{min}}$ are respectively the maximum and minimum distances to be covered), we can avoid solutions which imply velocities smaller than $v_{\text{min}}$.

The time optimal path to reach the waypoint on the considered operation space will be composed of a curve, done with maximum actuation, (not a perfect arc of a circle in the inertial frame, due to the effect of the disturbance), followed by a line segment, also done at maximum velocity. The arguments for the existence of a single curve are the same from [5], noting that our considerations about efficient trajectories avoid crossing the discontinuities of the value function.

B. Goto maneuver controller

This section describes the maneuver controller responsible by the execution of the goto maneuver. The objective of this maneuver is to drive the vehicle from its current position $(x_0, y_0)$ to the target waypoint $(x_w, y_w)$ in $T$ seconds. Since the termination time $T$ of the maneuver is already known at this step, the next logical step is to minimize the power consumption of the vehicles. For that purpose, consider the following optimal control problem:

$$\min \int_0^T v^3(t) dt \quad (11)$$

subject to:

$$x(t)' = v(t) \cos(\psi(t)) + v_{dx} \quad (12)$$

$$y(t)' = v(t) \sin(\psi(t)) + v_{dy} \quad (13)$$

$$v_{\text{min}} \leq v(t) \leq v_{\text{max}} \quad (14)$$

$$[x(0) \ y(0) \ x(T) \ y(T)]^T = [x_0 \ x_w] \quad (15)$$

Since the result is generalizable for all vehicles, the formulation is done for single vehicle to avoid cluttering. For slow varying velocities, $\int_0^T v^3(t) dt$ is a good indicator of the power consumption of the AUV over period $[0, T]$.

The analysis of the optimal control problem 11 shows that under a given constant disturbance, $v(t)$ and $\psi(t)$ should be kept constant (with the derived optimal values) over the considered period and that the optimal path is a line segment joining the two waypoints. Basically, the solution will correspond to a constant velocity trajectory along a straight line.

Actually the same decoupling could be also applied for the case where no assignment is provided. That would imply the calculation of the minimum time trajectory from each vehicle’s initial location to each target waypoint and choosing the combination with the smallest maximum travelling time. However, we do not develop that scenario on this paper.

The expression is based on the energy spent to counteract the drag force $Br(t)v(t)$; in fact, the expression should be $\int_0^T [v(t)]^2 dt$ but, since $v(t)$ is always nonnegative, the expressions are equivalent.
line segment joining the vehicle’s initial position, translated by the offset vector caused by the disturbance during time $T$, and the waypoint. The obtained path is the same that it would be obtained for the minimum time problem. Notice also that, unless the disturbance has the same orientation of the line segment joining the two waypoints, $\psi(t)$ will be different from the orientation of the line segment.

The above formulation contains a strong simplification on the turning dynamics since it puts no constraints on the variation rate of the vehicle’s orientation. The result found above would require the initial orientation of the vehicle to be the optimal one, since obviously it is not possible to change the vehicle’s orientation in null time. However, this result provides insight for the approach described below.

If the dynamic constraints on $\psi(t)$ are considered, the analysis shows that the trajectory will consist of an arc (obtained with maximum turning actuation) followed by a line segment. Since $\psi(t)$ must be continuous, the line segment must be tangent to the arc. It must be remarked that this result is valid only for sufficiently large distances, as defined in section III.

The underlying idea is, instead of solving analytically the complete optimal control problem, to use simple geometrical considerations (see figure 4): since the final time of the trajectory is known a priori, the total offset $d_{dT}$ to be caused by the disturbance during this time is calculated and added to the vehicle’s initial position $x_0$. From now on, the system can be treated like the typical Dubins vehicle (equation 1 with $v_d = 0$). The necessary angular displacement for the vehicle to be facing the target waypoint $x_w$ is calculated. The vehicle uses maximum angular velocity for turning, originating a circular trajectory with radius $r_{\text{min}}$ and centered such as the vehicle’s initial posture is tangent to this trajectory (see equations below, namely $d_{\text{wd}}$).

Thus, after each request for the execution of a goto maneuver (startGoto event), the maneuver controller calculates which way (clockwise or counterclockwise) the vehicle should turn in order to be facing the waypoint (in the “compensated” coordinate system), the orientation $\psi_{\text{turn}}$ which will have to be acquired by the vehicle before entering the straight line trajectory to the waypoint and the desired velocity $v$. The velocity is simply given by the quotient between the length of the path and the desired termination time. The maneuver controller generates a trajectory for a virtual vehicle, modelled by the kinematic model of equation 1, whose posture is tracked by the tracking law of the vehicle. The expressions for $\psi_{\text{turn}}$ and $v$ are given below and illustrated on figure 4.

\[
\psi_{\text{turn}} = \begin{cases} 
\pm \left( \pi - \arctan \left( \frac{r_{\text{min}}}{\sqrt{\|d_{\text{wd}}\|^2 - r_{\text{min}}^2}} \right) \right) \\
+ \alpha_{\text{wdr}}
\end{cases}
\]  

Equation 16 is obtained by considering $x_w$ as the new origin and $d_{\text{wd}}$ as the new x axis and then finding the turning point $(x'_{\text{turn}}, y'_{\text{turn}})$ where the slope of the circle is equal to the segment connecting that point to the origin. The term $\arctan \left( \frac{r_{\text{min}}}{\sqrt{\|d_{\text{wd}}\|^2 - r_{\text{min}}^2}} \right)$ is derived from the slope of the line segment connecting the origin with the turning point. The length of the path, used on equation 17, is simply the sum of the length of the line segment with the length of the arc that compose the trajectory. For the sake of completeness:

\[
x'_{\text{turn}} = \frac{\|d_{\text{wd}}\|^2 - r_{\text{min}}^2}{\|d_{\text{wd}}\|^2} \\
y'_{\text{turn}} = r_{\text{min}} \frac{\sqrt{\|d_{\text{wd}}\|^2 - r_{\text{min}}^2}}{\|d_{\text{wd}}\|^2}
\]

The plus/minus sign is chosen accordingly to the initial turning direction (plus for counterclockwise, minus for clockwise). The turning direction, $\omega$, is determined

\[
v = \sqrt{\|d_{\text{wd}}\|^2 + r_{\text{min}}^2 - \|d_{\text{wd}}\|^2} \left( \frac{r_{\text{min}}}{\|d_{\text{wd}}\|^2} \right) + r_{\text{min}} |\psi_{\text{turn}} - \psi_0| T^{-1}
\]
The ideal trajectory should be travelled at 1 m/s, perpendicular to the ideal trajectory (line segment connecting the initial and final waypoints). The ideal trajectory should be travelled at 1 m/s using the following rule:

\[
\text{sign}(\omega) = \begin{cases} 
-1, & \delta_y > 0 \\
1, & \delta_y < 0 
\end{cases}
\]

with

\[
\delta_y = \frac{0}{\|d_{wd}\|} \begin{bmatrix} x_{wd} & y_{wd} \end{bmatrix} \begin{bmatrix} \cos(\psi_d) \\ \sin(\psi_d) \end{bmatrix}
\]

and \(d_{wd} = (x_{wd}, y_{wd})\). The disturbance \((v_{dx}, v_{dy})\) can be measured a priori or estimated on-line using a scheme such as the one described in [10].

The vehicle reaches the waypoint with an accuracy which is bounded by the sensing capabilities and the inherent control limitations. To overcome this limitation, when a vehicle reaches a neighborhood of the waypoint with radius \(r_{tol}\), the waypoint is considered reached. The state equations for the trajectory are:

\[
\begin{align*}
x'_{\text{ref}} &= v \cos(\psi_{\text{ref}}) + v_{dx} \\
y'_{\text{ref}} &= v \sin(\psi_{\text{ref}}) + v_{dy} \\
\psi'_{\text{ref}} &= \begin{cases} 
\text{sign}(\omega) \frac{r_{min}}{v_{min}}, & \text{if turning;} \\
0, & \text{if straight line.}
\end{cases}
\]

For most scenarios the approach described in this section will be more efficient than just trying to track a straight line trajectory connecting the two waypoints and moreover it is more compliant with the vehicle’s motion constraints. Figure 5 shows both type of trajectories for the undisturbed vehicle. For linear systems with full actuation it is more compliant with the vehicle’s motion constraints. The simple line tracking trajectory will be more efficient than just trying to track a straight line.

It must be remarked that the the maneuver controller is designed under the assumption that the target waypoints are chosen such that the computed \(v\) for each vehicle is not less than \(v_{min}\). For applications with large disparities between the distances to be covered by each vehicle, as discussed in the previous subsection, other solutions would have to be employed.

The maneuver controller was implemented as a C library and was tested in our simulation framework in a multi-phase operation scenario similar to the one described in [9]. The simulations use the nonlinear model of the AUV.

V. CONCLUSIONS

It was verified that the bounds derived with the kinematic model provide efficient trajectories when applied to the intrinsically nonlinear model of the AUVs. The trajectories generated using the kinematic model proved to be suitable for the intended operation in spite of the discontinuities on the angular speed.

The general approach presented in this work can be applied to other types of vehicles. Obviously, the main bottleneck is the calculation of the optimal trajectories for each considered vehicle. For linear systems with full actuation this is relatively easy. However, if other nonlinear models are considered a careful analysis is required.

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